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Title: CONTINUITY AND DIFFERENTIABILITY OF FUNCTION

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CONTINUITY AND DIFFERENTIABILITY OF FUNCTION

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Abstract. This article describes the concept of function, the concept of continuity, differential, types of functions, non-continuous function, their causes

Keywords. function, continuous function, function differential, their types

Introduction.

The property of continuity is exhibited by various aspects of nature. The water flow in the rivers is continuous. The flow of time in human life is continuous i.e. you are getting older continuously. And so on. Similarly, in mathematics, we have the notion of the continuity of a function.

In mathematics, a continuous function is a function that does not have any abrupt changes in value, known as discontinuities. More precisely, a function is continuous if arbitrarily small changes in its output can be assured by restricting to sufficiently small changes in its input. If not continuous, a function is said to be discontinuous. Up until the 19th century, mathematicians largely relied on intuitive notions of continuity, during which attempts such as the epsilon–delta definition were made to formalize it.

Continuity of functions is one of the core concepts of topology, which is treated in full generality below. The introductory portion of this article focuses on the special case where the inputs and outputs of functions are real numbers. A stronger form of continuity is uniform continuity. As an example, the function H(t) denoting the height of a growing flower at time t would be considered continuous. In contrast, the function M(t) denoting the amount of money in a bank account at time t would be considered discontinuous, since it "jumps" at each point in time when money is deposited or withdrawn.

What it simply means is that a function is said to be continuous if you can sketch its curve on a graph without lifting your pen even once (provided that you can draw good). It is a very straightforward and close to accurate definition actually. But for the sake of higher mathematics, we must define it in a more precise way.

Definition of Continuity

A function f(x) is said to be continuous at a point x = a, in its domain if the following three conditions are satisfied:

f(a) exists (i.e. the value of f(a) is finite)

Limx→a f(x) exists (i.e. the right-hand limit = left-hand limit, and both are finite)

Limx→a f(x) = f(a)

The function f(x) is said to be continuous in the interval I = [x1,x2] if the three conditions mentioned above are satisfied for every point in the interval I.

However, note that at the end-points of the interval I, we need not consider both the right-hand and the left-hand limits for the calculation of Limx→a f(x). For a = x1, only the right-hand limit need be considered, and for a = x2, only the left-hand limit needs to be considered.

Topics under Continuity And Differentiability

Algebra of Continuous Functions

Algebra of Derivatives

Derivatives of Composite Functions

Derivatives of Functions in Parametric Forms

Derivatives of Implicit Functions

Derivatives of Inverse Trigonometric Functions

Exponential and Logarithmic Functions

Logarithmic Differentiation

Mean Value Theorem

Second Order Derivatives

Some Typical Continuous Functions

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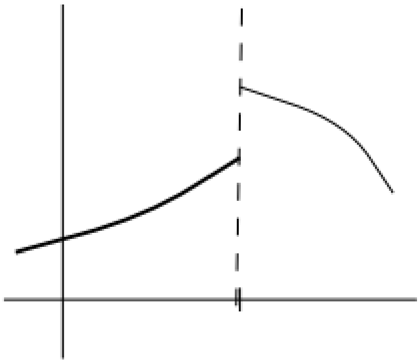
Trigonometric Functions in certain periodic intervals (sin x, cos x, tan x etc.)
Polynomial Functions ($x2 + x + 1$, $x4 + 2$….etc.)
Exponential Functions ($e2x$, $5ex$ etc.)
Logarithmic Functions in their domain ($\log 10x$, $\ln x2$ etc.)

Discontinuity

If any one of the three conditions for a function to be continuous fails; then the function is said to be discontinuous at that point. On the basis of the failure of which specific condition leads to discontinuity, we can define different types of discontinuities.

Jump Discontinuity

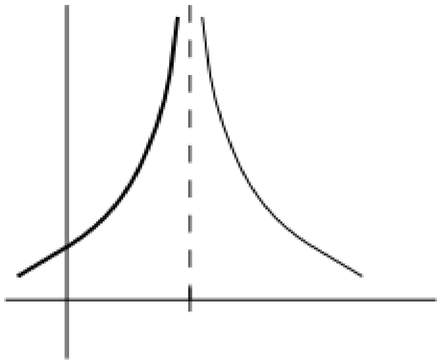
In this type of discontinuity, the right-hand limit and the left-hand limit for the function at x = a exists; but the two are not equal to each other. It can be shown as:$\mathrm{Lim}x\to a+f(x)\neq \mathrm{Lim}x\to a-f(x)\mathrm{Lim}x\to a+f(x)\neq \mathrm{Lim}x\to a-f(x)$



jump

Infinite Discontinuity

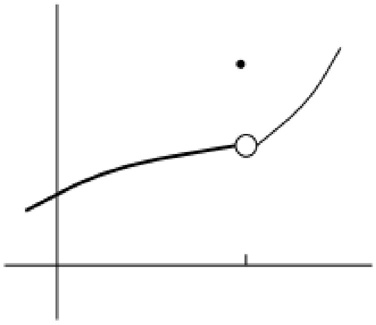
The function diverges at x = a to give it a discontinuous nature here. That is to say, f(a) is not defined f(a) is not defined Since the value of the function at x = a tends to infinity or doesn't approach a particular finite value, the limits of the function as $x \to a$ are also not defined.



infinite

Point Discontinuity

This is a category of discontinuity in which the function has a well defined two-sided limit at x = a, but either f(a) is not defined or f(a) is not equal to its limit. The discrepancy can be shown as:$\mathrm{Lim}x\to af(x)\neq f(a)\mathrm{Lim}x\to af(x)\neq f(a)$This type of discontinuity is also known as a Removable Discontinuity since it can be easily eliminated by redefining the function in such a way that,$f(a)=\mathrm{Lim}x\to af(x)f(a)=\mathrm{Lim}x\to af(x)$



removable

Question 1: Let a function be defined as f(x) =
$5 – 2x$ for $x < 1$
3 for $x = 1$
$x + 2$ for $x > 1$
Is this function continuous for all x?

Answer : Since for $x < 1$ and $x > 1$, the function f(x) is defined by straight lines (that can be drawn continuously on a graph), the function will be continuous for all $x \neq 1$. Now for $x = 1$, let us check all the three conditions:
–> f(1) = 3 (given)

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–> Left-Hand Limit:

=Limx→1−f(x)=Limx→1−f(x)

=Limx→1−(5–2x)=Limx→1−(5–2x)

=5–2×1=5–2×1

=3=3

–> Right-Hand Limit:

=Limx→1+f(x)=Limx→1+f(x)

=Limx→1+(x+2)=Limx→1+(x+2)

=1+2=1+2

=3=3

Limx→1−f(x)=Limx→1+f(x)=3=f(1)Limx→1−f(x)=Limx→1+f(x)=3=f(1)

Thus all the three conditions are satisfied and the function f(x) is found out to be continuous at x = 1. Therefore, f(x) is continuous for all x.

This concludes our discussion on the topic of continuity of functions. Continuous functions are very important as they are necessarily differentiable at every point on which they are continuous, and hence very simple to work upon.

The three conditions of continuity are as follows:

The function is expressed at x = a.

The limit of the function as the approaching of x takes place, a exists.

The limit of the function as the approaching of x takes place, a is equal to the function value f(a).

A limit refers to a number that a function approaches as the approaching of an independent variable of the function takes place to a given value. For example, given the function f (x) = 3x, the limit of f (x) as the approaching of x takes place to 2 is 6. Symbolically, one can write this as f (x) = 6.

Discontinuous functions are those that are not a continuous curve. In a removable discontinuity, one can redefine the point so as to make the function continuous by matching the particular point’s value with the rest of the function.

There are several different definitions of continuity of a function. Sometimes a function is said to be continuous if it is continuous at every point in its domain. In this case, the function f(x) = tan(x), with the domain of all real $x \neq (2n+1)\pi/2$, n any integer, is continuous. Sometimes an exception is made for boundaries of the domain. For example, the graph of the function $f(x) = \sqrt{x}$, with the domain of all non-negative reals, has a left-hand endpoint. In this case only the limit from the right is required to equal the value of the function. Under this definition f is continuous at the boundary x = 0 and so for all non-negative arguments. The most common and restrictive definition is that a function is continuous if it is continuous at all real numbers. In this case, the previous two examples are not continuous, but every polynomial function is continuous, as are the sine, cosine, and exponential functions. Care should be exercised in using the word continuous, so that it is clear from the context which meaning of the word is intended.

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7. Speck, Jared (2014). "Continuity and Discontinuity" (PDF). MIT Math. p. 3. Retrieved 2016-09-02. Example 5. The function 1/x is continuous on $(0, \infty)$ and on $(-\infty, 0)$, i.e., for $x > 0$ and for $x < 0$, in other words, at every point in its domain. However, it is not a continuous function since its domain is not an interval. It has a

single point of discontinuity, namely $x = 0$,
and it has an infinite discontinuity there.