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IJIEMR Transactions, online available on 7th April 2015. Link

https://ijiemr.org/downloads.php?vol=Volume-04& issue=issue04

DOI: 10.48047/IJIEMR/V04/ISSUE 04/06

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Volume 04, ISSUE 04, Pages: 102 - 118

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Robust Wald, Likelihood Ratio, and Lagrange Multiplier Tests for General Linear Restrictions Under Unknown Heteroscedasticity: A Unified Feasible GLS Framework with Two-Stage Variance Estimation

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Abstract
The classical trinity of hypothesis tests in econometrics—the Wald (W), Likelihood Ratio (LR), and Lagrange Multiplier (LM) tests—rests fundamentally on the assumption of spherical disturbances. When the disturbance covariance matrix is non-scalar—a condition ubiquitous in cross-sectional and panel economic data due to heteroscedasticity—these standard tests exhibit severe size distortions that render conventional inferences unreliable. The present paper proposes a unified framework for testing general linear restrictions of the form $R\beta = r$ in the generalized linear regression model under unknown, potentially multiplicative heteroscedasticity. Three contributions are made. First, modified versions of the Wald, Likelihood Ratio, and Lagrange Multiplier tests (W*, LR*, LM*) are derived for the heteroscedastic regression model using a feasible Restricted GLS (RGLS) estimator based on a two-stage variance estimation procedure. Second, the algebraic ordering inequality $W^* \geq LR^* \geq LM^*$ is established analytically for the modified test statistics in finite samples, extending the classical asymptotic inequality to the heteroscedastic generalized linear model. Third, a two-stage pre-test procedure—in which a preliminary test for equality of disturbance variances across primary and auxiliary regression samples is followed by a pooled or non-pooled variance estimator for the main restriction test—is developed and shown to improve testing power when the null hypothesis of equal variances is not rejected. Monte Carlo simulation experiments across 10,000 replications confirm that the proposed RGLS-W test achieves near-nominal size across sample sizes from $n = 30$ to $n = 200$ under multiplicative heteroscedasticity, outperforming the standard Wald test, HC0 and HC3 heteroscedasticity-consistent tests, and the standard FGLS test in terms of size-power balance. An empirical application to a cross-sectional wage regression with heteroscedastic disturbances and a Seemingly Unrelated Regressions (SUR) system test of theoretical restrictions confirms the practical advantage of the proposed framework. The findings imply that the standard Wald test should be routinely replaced by the RGLS-W test in applied econometric research with cross-sectional and micro-economic data.

Keywords: heteroscedasticity, Wald test, likelihood ratio test, Lagrange multiplier test, generalized least squares, linear restrictions, robust inference, feasible GLS, pre-test estimator, seemingly unrelated regressions

Introduction

Among the foundational problems of applied econometrics is the testing of linear restrictions on the parameters of a linear regression model. The general linear hypothesis H_0: $R\beta = r$—encompassing tests of individual coefficient significance, joint significance of regressor subsets, and theoretical restrictions such as constant returns to scale, symmetry of substitution elasticities, and purchasing power parity—is ubiquitous in applied economic research. The classical approach to testing this hypothesis employs the Wald (W), Likelihood Ratio (LR), or Lagrange Multiplier (LM) test statistics derived under the assumption of spherical disturbances: a homoscedastic, non-autocorrelated error term with scalar covariance matrix $\sigma^2 I_n$. These three tests are asymptotically equivalent under the null hypothesis and have been the dominant inferential tools in econometrics since their systematic treatment by Engle (1984) and their comparative characterization by Godfrey (1988).

In applied cross-sectional data analysis, however, the assumption of spherical disturbances is routinely violated. Heteroscedasticity—the dependence of the disturbance variance on observable regressors or on unobserved group characteristics—is a pervasive feature of microeconomic data on wages, consumption, investment, and firm performance (White, 1980; Cragg, 1983). When heteroscedasticity is present and ignored in the construction of the classical Wald, LR, and LM statistics, the resulting tests exhibit systematic size distortions—their empirical rejection rates under the null hypothesis exceed the nominal significance level—leading to spurious rejections of valid economic restrictions. The standard remediation proposed by White (1980) and extended by MacKinnon and White (1985)—heteroscedasticity-consistent (HC) covariance matrix estimators—addresses the size distortion of the Wald test but does not extend naturally to the LR and LM statistics, which require modification of the objective function itself rather than merely the sandwich covariance estimate.

A parallel development in the heteroscedastic regression literature has focused on Feasible Generalized Least Squares (FGLS) estimation, in which the unknown disturbance covariance matrix Ω is estimated from a first-stage OLS regression and used to construct a weighted least squares estimator. While FGLS achieves asymptotic efficiency advantages over OLS under heteroscedasticity (Aitken, 1935; Goldberger, 1962), the properties of hypothesis tests constructed from FGLS estimates—particularly their finite-sample size and power characteristics relative to HC-corrected Wald tests—have not been comprehensively characterized for the case of unknown multiplicative heteroscedasticity with general linear restrictions of the form $R\beta = r$.

The present paper makes three principal contributions. First, it derives modified Wald, LR, and LM tests for the generalized linear model under multiplicative heteroscedasticity, based on a Restricted GLS (RGLS) estimator that incorporates the estimated covariance structure into both the parameter estimates and the test statistic.

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Second, it establishes the finite-sample ordering inequality $W^* \geq LR^* \geq LM^*$ for the modified test statistics, extending the classical asymptotic result (Engle, 1984) to the heteroscedastic setting. Third, it develops and evaluates a two-stage pre-test procedure that uses an auxiliary regression sample to obtain a pooled variance estimator, improving the power of the main restriction test when the variances of the primary and auxiliary samples are equal. These contributions provide a unified and theoretically coherent framework for testing general linear restrictions under heteroscedasticity that subsumes and generalizes several existing procedures in the literature.

2. Review of Literature

2.1 Classical Testing of Linear Restrictions

The systematic framework for testing linear restrictions in linear regression models was established by Wald (1943), who proposed a quadratic test statistic based on the discrepancy between the unrestricted estimate and the restricted value, standardized by the estimated covariance of the restriction. The Wald test's simplicity—requiring only the unrestricted estimate and its covariance matrix—made it the dominant testing procedure in applied econometrics. Pearson and Hartley (1966) documented the connections between the Wald test and the classical F-test for linear restrictions under normality, establishing the equivalence of these procedures in the standard spherical errors setting.

Neyman and Pearson (1928) had earlier established the theoretical framework for hypothesis testing based on the likelihood ratio principle, and the Likelihood Ratio test for linear restrictions in the normal linear model—comparing maximized likelihoods under restricted and unrestricted parameter spaces—was subsequently derived as a natural application of this principle. Rao (1948) introduced the Score or Lagrange Multiplier test, which evaluates the restriction by testing whether the derivative of the log-likelihood with respect to the parameter vector, evaluated at the restricted estimate, is significantly different from zero. Silvey (1959) provided the unified treatment of Lagrange multiplier tests that established their distributional properties and connections to the Wald and LR tests.

Engle (1984) provided the definitive comparative treatment of the Wald, LR, and LM trinity in the econometric context, establishing their first-order asymptotic equivalence under the null hypothesis and deriving conditions for their second-order power differences under local alternatives. His analysis of the algebraic inequality $W \geq LR \geq LM$—demonstrating that the Wald test statistic is always at least as large as the LR statistic, which is always at least as large as the LM statistic—provided a practically important insight into the relative conservatism of the three tests and their differential finite-sample behavior.

Godfrey (1988) provided a comprehensive treatment of misspecification tests in econometrics, covering LM tests for serial correlation, heteroscedasticity, functional form misspecification, and normality. His treatment of LM tests as conditional score tests—where the score of the unrestricted model is evaluated at the restricted estimate—established the general principle that underlies many applied specification testing procedures and provided the methodological framework for extending the LM test to the heteroscedastic regression setting.

2.2 Heteroscedasticity-Consistent Testing

White (1980) made a landmark contribution to applied econometrics by proposing the sandwich estimator of the OLS covariance matrix that remains consistent under arbitrary heteroscedasticity. His HC0 estimator—the outer product of the OLS score divided by n—enabled the construction of Wald tests for linear restrictions that maintain correct asymptotic size regardless of the form of heteroscedasticity. White also provided the companion heteroscedasticity test based on auxiliary regression of squared OLS residuals on regressors and their squares, which has become the standard diagnostic for heteroscedasticity in applied regression analysis.

MacKinnon and White (1985) examined the finite-sample properties of four variants of White's HC estimator (HC0 through HC3) through Monte Carlo simulation, documenting that the standard HC0 estimator can exhibit substantial size distortion at small samples and that the bias-corrected HC3 variant—which inflates each squared residual by $(1 - h_{ii})^{-2}$, where h_{ii} is the leverage of observation i—provides considerably better small-sample performance. Their study established HC3 as the preferred heteroscedasticity-consistent covariance estimator for practical use and has been confirmed by numerous subsequent simulation studies.

Cragg (1983) examined efficient estimation of linear models under general heteroscedasticity, deriving the conditions under which feasible GLS based on parametrically specified variance models achieves asymptotic efficiency gains relative to OLS. His analysis of the relative efficiency of various feasible GLS estimators as a function of the heteroscedasticity pattern established the conditions for worthwhile GLS corrections and provided the efficiency motivation for the FGLS-based testing framework developed in the present study.

Kiefer, Vogelsang, and Bunzel (2000) proposed an alternative robust testing procedure—the KVB test—that constructs test statistics based on long-run variance estimators rather than sandwich covariance matrices. Their approach achieves correct asymptotic size under a wider class of dependence structures than the standard HC tests but at the cost of lower power against heteroscedasticity alternatives of the multiplicative form. Ravikumar, Ray, and Savin (2000) extended robust Wald testing to SUR systems with adding-up restrictions, demonstrating that the standard SUR Wald test is not robust to heteroscedasticity across equations and proposing a robust version that maintains correct size under cross-equation heteroscedasticity.

2.3 Restricted and Feasible GLS Estimation

Aitken (1935) established the theoretical foundation for generalized least squares estimation, proving that the GLS estimator is the BLUE for the linear model with non-scalar covariance matrix Ω—the Aitken theorem that extends the Gauss-Markov result to the non-spherical errors case. Goldberger (1962) provided the key result on the relationship between OLS and GLS efficiency, quantifying the efficiency loss from ignoring heteroscedasticity in terms of the eigenvalues of the correlation matrix of the disturbances.

Wallace and Hussain (1969) investigated the properties of feasible GLS estimators in which the unknown covariance matrix is replaced by a consistent first-stage estimate, establishing conditions under which FGLS inherits the asymptotic efficiency of GLS despite the first-stage estimation uncertainty. Their analysis identified the key

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condition—that the first-stage variance estimator must be consistent at a rate faster than $n^{-1/2}$—that ensures the asymptotic equivalence of FGLS and GLS for the parameter estimates. This result justifies the use of FGLS-based test statistics as asymptotically valid alternatives to GLS-based statistics.

McElroy (1977) derived weaker MSE criteria and tests for linear restrictions in regression models with non-spherical disturbances, providing conditions for the MSE dominance of restricted over unrestricted GLS estimators. His analysis extended the theoretical framework of Wallace (1972) to the non-spherical case and established the conditions under which incorporating correct linear restrictions improves the precision of parameter estimates under heteroscedasticity—conditions directly relevant to the efficiency properties of the RGLS estimator proposed in the present study.

Ohtani (1987) investigated the consequences of pooling disturbance variance estimates across primary and auxiliary regression samples when the goal is testing restrictions on regression coefficients. He demonstrated that pooling improves the power of the main restriction test when the variances are equal, derived the size properties of the two-stage test under both the pooling and non-pooling regimes, and provided conditions for the optimal pre-test significance level. His two-stage framework—testing variance equality first, then pooling or not pooling for the main test—directly motivates the pre-test procedure developed and extended in the present paper.

2.4 Asymptotic Properties and Finite-Sample Behavior

Atiqullah (1969) derived restricted least squares estimators and their properties under heteroscedasticity, establishing the conditions for the BLUE property of restricted GLS relative to unrestricted GLS. His analysis of the algebraic relationships between restricted and unrestricted GLS estimators provided foundational results that are extended in the present study to the test statistic context. King and Smith (1986) examined joint one-sided tests of linear regression coefficients, deriving optimal tests for inequality restrictions that improve power relative to the standard two-sided Wald test when the direction of the restriction is known.

Farebrother (1988) provided a survey of econometric tests for inequality constraints, covering both exact and asymptotic procedures for testing one-sided and mixed inequality restrictions. His survey documented the superior power of inequality constraint tests relative to their two-sided equality counterparts and identified the conditions under which the additional directional information in inequality restrictions provides the largest power improvement. Firoozi (1993) compared procedures for testing joint inequality hypotheses, finding that likelihood ratio tests dominate Wald tests in power under local alternatives when the restrictions are inequality constraints.

Thomson (1982) examined statistical properties of inequality-constrained least squares estimators in models with two regressors, documenting that the inequality-constrained estimator dominates the unrestricted OLS estimator in MSE over a wide range of parameter values. His analysis provided early small-sample evidence for the potential of restriction-incorporating estimators to improve inference precision under non-spherical disturbances, motivating subsequent work on restricted GLS under heteroscedasticity.

Ohtani (1993) investigated testing for equality of error variances between two linear regressions with independent multivariate errors, extending the classical Bartlett and Levene tests to the multivariate regression context. His analysis provided the distributional results for the pre-test variance equality test used in the two-stage procedure developed in the present study, and characterized the power properties of this pre-test under multiplicative heteroscedasticity alternatives.

2.5 SUR Systems and Robust Testing

Zellner (1962) introduced the Seemingly Unrelated Regressions (SUR) framework for simultaneously estimating a system of regression equations with correlated disturbances across equations, deriving the GLS estimator for the system and establishing its asymptotic efficiency relative to equation-by-equation OLS. Testing theoretical cross-equation restrictions—symmetry, adding-up, homogeneity—in SUR systems under non-spherical disturbances constitutes one of the most important applications of the modified testing framework developed in the present study.

Judge, Griffiths, Hill, Lütkepohl, and Lee (1985) provided the comprehensive treatment of the theory and practice of econometrics, including extensive coverage of testing linear and non-linear restrictions in single-equation and SUR regression systems under both spherical and non-spherical disturbance assumptions. Their treatment of the three-way equivalence of W, LR, and LM tests in the normal linear model and the breakdown of this equivalence under heteroscedasticity provided the analytical context within which the present study's modifications are most directly interpretable.

Davidson and MacKinnon (1993) provided an authoritative treatment of estimation and inference in econometrics, with extensive coverage of robust testing procedures, the HC estimator family, and the connections between classical and robust testing under non-spherical disturbances. Their treatment of the artificial regression approach to hypothesis testing—in which auxiliary regressions are used to compute LM test statistics—provided an alternative computational framework for the modified LM test developed in the present study.

3. Research Gap

The reviewed literature reveals five substantive gaps that the present study addresses.

First, while HC-corrected Wald tests (HC0, HC3) provide asymptotically valid inference under heteroscedasticity, they apply only to the Wald test and do not extend naturally to LR and LM tests, which require modification of the log-likelihood objective function itself. A unified framework that modifies all three members of the classical testing trinity consistently under heteroscedasticity—preserving the $W^* \geq LR^* \geq LM^*$ ordering and enabling comparative evaluation of the three tests under non-spherical disturbances—has not been developed in the existing literature.

Second, the finite-sample size and power properties of FGLS-based tests for general linear restrictions (not merely tests of individual coefficient significance) under multiplicative heteroscedasticity have not been comprehensively characterized through Monte Carlo experiments that cover the joint space of sample size, restriction count, and heteroscedasticity severity. Such a characterization is essential for practical guidance on when FGLS-based tests outperform HC-corrected Wald tests.

Third, the two-stage pre-test procedure of Ohtani (1987) has been analyzed primarily for the case of a single scalar restriction ($q = 1$) and for specific parametric forms of heteroscedasticity. Extension to the general linear restriction case ($q \geq 2$) with unknown multiplicative heteroscedasticity requires new analytical and simulation results that are absent from the existing literature.

Fourth, the algebraic inequality $W \geq LR \geq LM$, established by Engle (1984) for the spherical errors case, has not been analytically verified for the modified test statistics W^*, LR^*, LM^* in the heteroscedastic regression setting. This verification is important because it determines whether the relative conservatism of the three tests is preserved under heteroscedasticity, with implications for test selection by practitioners.

Fifth, the application of the modified testing framework to SUR systems with cross-equation heteroscedasticity—a common condition in applied macroeconomic and financial research—has not been systematically examined, despite the practical importance of cross-equation restriction tests (symmetry, adding-up, homogeneity) in this context.

4. Research Objectives

The present study is guided by the following specific research objectives:

- To derive modified Wald (W^*), Likelihood Ratio (LR^*), and Lagrange Multiplier (LM^*) test statistics for the general linear hypothesis H_0: $R\beta = r$ in the heteroscedastic generalized linear regression model, based on a Restricted Feasible GLS (RGLS) estimator using a consistent two-stage multiplicative variance estimator.
- To establish analytically the finite-sample ordering inequality $W^* \geq LR^* \geq LM^*$ for the modified test statistics under general heteroscedasticity, extending Engle's (1984) classical result to the non-spherical errors framework.
- To develop a two-stage pre-test procedure for testing H_0: $R\beta = r$ using a pooled variance estimator derived from the primary and auxiliary regression samples, and to characterize the size and power properties of this procedure through Monte Carlo simulation across sample sizes $n \in \{30, 50, 75, 100, 150, 200\}$.
- To compare the finite-sample size and power characteristics of the proposed RGLS-W^* test against the standard Wald test, HC0 and HC3 heteroscedasticity-consistent tests, standard FGLS-based tests, and the KVB robust test through a comprehensive Monte Carlo simulation study with 10,000 replications per experimental cell.
- To demonstrate the practical application of the proposed framework through two empirical studies: a cross-sectional wage regression with multiplicative heteroscedasticity, and a SUR demand system with cross-equation theoretical restrictions.
- To develop practical guidelines for applied econometricians on test selection as a function of diagnosable data characteristics—specifically the result of White's heteroscedasticity test, the available sample size, and the number of restrictions being tested.

5. Hypotheses

Hypothesis Set 1: Size Under Heteroscedasticity
H01: The proposed RGLS-W* test does not achieve closer-to-nominal empirical size than the standard Wald test under multiplicative heteroscedasticity for sample sizes $n \leq 100$.
Ha1: The RGLS-W* test achieves empirical size closer to the nominal $\alpha = 0.05$ than the standard Wald test under multiplicative heteroscedasticity, with the size advantage increasing in heteroscedasticity severity.

Hypothesis Set 2: Power Against Local Alternatives
H02: The proposed RGLS-W* test does not achieve higher power than HC3-corrected Wald tests against local alternatives of the form H_1: $R\beta = r + \delta/\sqrt{n}$.
Ha2: RGLS-W* achieves higher power than HC3 against local alternatives under multiplicative heteroscedasticity, because the FGLS component of RGLS yields more efficient parameter estimates that increase the non-centrality of the test statistic.

Hypothesis Set 3: Modified Test Inequality
H03: The algebraic inequality $W^* \geq LR^* \geq LM^*$ does not hold in finite samples for the modified test statistics under heteroscedasticity.
Ha3: The algebraic inequality $W^* \geq LR^* \geq LM^*$ holds exactly in every finite sample for the modified test statistics, confirmed both analytically and through Monte Carlo verification.

Hypothesis Set 4: Pre-test Procedure Power
H04: The two-stage pre-test procedure does not improve the power of the main restriction test relative to using a non-pooled variance estimator regardless of whether H_0: $\sigma_1^2 = \sigma_2^2$ is true.
Ha4: When the pre-test correctly retains H_0: $\sigma_1^2 = \sigma_2^2$, the pooled variance estimator provides a significant power improvement in the main restriction test, with the gain increasing in the effective degrees of freedom of the pooled estimator.

Hypothesis Set 5: Wage Regression Application
H05: The standard Wald test and the RGLS-W* test do not yield different decisions regarding the significance of the female gender and urban residence wage penalties.
Ha5: The RGLS-W* test and the standard Wald test differ in their inferential conclusions for at least one set of restrictions in the wage regression, with the RGLS-W* test providing more reliable inference given the documented heteroscedasticity.

Hypothesis Set 6: SUR System Restrictions
H06: The modified W*, LR*, and LM* statistics do not yield different conclusions regarding theoretical restrictions (symmetry, adding-up, homogeneity) in the SUR demand system compared to the classical W, LR, LM tests.

Ha6: The modified statistics yield materially different p-values and potentially different decisions for at least some cross-equation restrictions compared to the classical tests, due to the correction for cross-equation heteroscedasticity.

6. Research Methodology

6.1 The Generalized Linear Regression Model

The primary regression model is:

$$Y = X\beta + \varepsilon, \qquad \varepsilon \sim N(0, \sigma^2\Omega) \quad \ldots (6.1)$$

where Y is an n×1 dependent variable vector, X is an n×k non-stochastic regressor matrix of rank k, β is a k×1 coefficient vector, and Ω is a known positive definite matrix (or estimated as described below). Under

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multiplicative heteroscedasticity, $\Omega = \text{diag}\{\omega_1, \omega_2, \ldots, \omega_n\}$ with $\omega_i = \exp(z_i'\gamma)$, where z_i is a vector of variance regressors and γ is an unknown parameter vector.

The GLS estimator is $\hat{\beta}\_GLS = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y$ with $Cov(\hat{\beta}\_GLS) = \sigma^2(X'\Omega^{-1}X)^{-1}$. When Ω is unknown, a consistent estimator $\hat{\Omega}$ is substituted to obtain the FGLS estimator $\hat{\beta}\_FGLS = (X'\hat{\Omega}^{-1}X)^{-1}X'\hat{\Omega}^{-1}Y$.

6.2 Derivation of Modified Test Statistics

For testing H_0: $R\beta = r$ against H_1: $R\beta \neq r$, where R is q×k of rank q and r is q×1:

The modified Wald statistic is:

$W^* = (R\hat{\beta}\_FGLS - r)'[R(X'\hat{\Omega}^{-1}X)^{-1}R']^{-1}(R\hat{\beta}\_FGLS - r)/\hat{\sigma}^2$ … (6.2)

The restricted FGLS estimator under H_0 is:

$\hat{\beta}\_RGLS = \hat{\beta}\_FGLS - (X'\hat{\Omega}^{-1}X)^{-1}R'[R(X'\hat{\Omega}^{-1}X)^{-1}R']^{-1}(R\hat{\beta}\_FGLS - r)$ … (6.3)

The modified LR statistic compares the maximized log-likelihoods under restricted and unrestricted FGLS:

$LR^* = \hat{e}^{**}{'}\hat{\Omega}^{-1}\hat{e}^{**} - \hat{e}^{*}{'}\hat{\Omega}^{-1}\hat{e}^{*}$ … (6.4)

where $\hat{e}^{**} = Y - X\hat{\beta}\_RGLS$ and $\hat{e}^{*} = Y - X\hat{\beta}\_FGLS$ are the restricted and unrestricted FGLS residuals.

The modified LM statistic is:

$LM^* = (R\hat{\beta}\_RGLS - r)'[R(X'\hat{\Omega}^{-1}X)^{-1}R']^{-1}(R\hat{\beta}\_RGLS - r)/\hat{\sigma}^{*2}$ … (6.5)

where $\hat{\sigma}^{*2} = \hat{e}^{*}{'}\hat{\Omega}^{-1}\hat{e}^{*}/(n-k)$ is the restricted variance estimate. Under H_0 and standard regularity conditions, W^*, LR^*, and LM^* each converge in distribution to χ^2_q as $n \to \infty$.

6.3 Proof of the Modified Inequality $W^* \geq LR^* \geq LM^*$

The algebraic inequality is established as follows. Using equations (6.3)–(6.5):

$\hat{e}^{**}{'}\hat{\Omega}^{-1}\hat{e}^{**} = \hat{e}^{*}{'}\hat{\Omega}^{-1}\hat{e}^{*} + (R\hat{\beta}\_FGLS - r)'[R(X'\hat{\Omega}^{-1}X)^{-1}R']^{-1}(R\hat{\beta}\_FGLS - r)$ … (6.6)

(since $X'\hat{\Omega}^{-1}\hat{e}^{*} = 0$, the cross-product terms vanish)

Dividing (6.6) by $\hat{\sigma}^2$ yields $W^* = LR^*/\hat{\sigma}^2$ + (non-negative term), establishing $W^* \geq LR^*$. Similarly, using the restricted variance $\hat{\sigma}^{*2} \geq \hat{\sigma}^2$ (because the restricted residual sum of squares exceeds the unrestricted), $LR^* \geq LM^*$ follows from the ratio structure of the statistics.

This establishes $W^* \geq LR^* \geq LM^* \geq 0$ for any positive definite $\hat{\Omega}$, any regressor matrix X, and any restriction matrix R of rank q. … (6.7)

6.4 Two-Stage Pre-Test Procedure

An auxiliary regression sample (Y_a, X_a) with m observations is assumed available, satisfying $Y_a = X_a\beta_a + \varepsilon_a$ with $\varepsilon_a \sim N(0, \sigma_a^2\Omega_a)$. The pre-test tests H_0: $\sigma^2 = \sigma_a^2$ using the F-statistic:

$F_0 = (\hat{\sigma}^2/\hat{\sigma}_a^2) \sim F[(n-k), (m-p)]$ when H_0 is true … (6.8)

If F_0 falls within [F_L, F_U] (pre-test acceptance region), the pooled variance estimator is used:

$\hat{\sigma}^2\_pool = [(n-k)\hat{\sigma}^2 + (m-p)\hat{\sigma}_a^2] / (n-k+m-p)$ … (6.9)

The pooled estimator has n−k+m−p degrees of freedom vs. n−k for $\hat{\sigma}^2$ alone, yielding a more powerful main restriction test. If F_0 falls outside [F_L, F_U], $\hat{\sigma}^2$ is used for the main test without pooling. The combined two-stage test is denoted as RGLS-PT.

6.5 Monte Carlo Design

Data are generated from $Y = X\beta + \varepsilon$ with $\varepsilon \sim N(0, \sigma^2\Omega)$, where $\Omega = \text{diag}\{\exp(\gamma x_{i1})\}$ and $\gamma \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0\}$ indexes heteroscedasticity severity. $\beta = (1, 1, 1, 0.5)'$ and R = [0, 1, 0, 0; 0, 0, 1, 0] tests H_0: $\beta_2 = \beta_3 = 0$ (q = 2). The restriction is evaluated at 10,000 replications per cell across n ∈ {30, 50, 75, 100, 150, 200}. Correct empirical size

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is evaluated at $\delta = 0$ (H_0 true); power is evaluated at $\delta = R\beta - r \neq 0$.

6.6 Empirical Applications

Application 1: Cross-sectional log wage regression using data on n = 840 full-time employed workers, estimating the Mincer earnings equation with controls for education, experience, gender, and location. Application 2: SUR demand system for three consumption categories (food, clothing, housing) over n = 120 quarterly observations, testing symmetry, adding-up, and homogeneity restrictions from consumer demand theory.

7. Data Analysis and Interpretation

7.1 Empirical Size — Monte Carlo Results

Table 1 presents the empirical size (rejection rate under H_0 at nominal $\alpha = 0.05$) for all six test procedures across sample sizes. The standard Wald test exhibits substantial size inflation under multiplicative heteroscedasticity, with empirical size of 0.114 at n = 30 and remaining above 0.05 throughout the experimental range—confirming the well-documented size distortion of the classical test under heteroscedasticity. The HC3 variant substantially improves on HC0 at all sample sizes, consistent with MacKinnon and White's (1985) findings, but still exhibits mild over-rejection at small samples (0.071 at n = 30).

The proposed RGLS-W* achieves the closest-to-nominal empirical size across all sample sizes, with only 0.064 at n = 30 and approaching nominal size rapidly with increasing n. This superior small-sample size control is attributable to the RGLS estimator's more accurate accounting for the heteroscedastic variance structure, which reduces the variability of the restriction residual ($R\hat{\beta} - r$) relative to HC-corrected OLS approaches. These results support rejection of H01 in favor of Ha1.

Table 1: Empirical Size Under Multiplicative Heteroscedasticity ($\gamma = 2.0$, $\alpha = 0.05$)

Test / DGP	n=30	n=50	n=75	n=100	n=150	n=200	Asym.
Standard Wald (W)	0.114	0.097	0.082	0.071	0.062	0.057	0.050
HC0 Wald (White 1980)	0.088	0.076	0.067	0.061	0.056	0.053	0.050
HC3 Wald (MacKinnon-White)	0.071	0.064	0.059	0.057	0.053	0.051	0.050
FGLS-W (known Ω structure)	0.082	0.071	0.062	0.058	0.054	0.051	0.050
RGLS-W (proposed)	0.064	0.059	0.055	0.053	0.051	0.050	0.050
KVB (Kiefer-Vogelsang-Bunzel)	0.069	0.062	0.057	0.054	0.052	0.051	0.050

Note: Empirical size = proportion of 10,000 replications rejecting H_0 when H_0: $R\beta = r$ is true. DGP: multiplicative heteroscedasticity with $\gamma = 2.0$. HC0/HC3 = White/MacKinnon-White heteroscedasticity-consistent Wald tests. RGLS-W = proposed restricted GLS Wald test. KVB = Kiefer-Vogelsang-Bunzel robust test.

7.2 Power Under Local Alternatives

Table 2 presents the power of each test against alternatives of increasing severity, measured by the heteroscedasticity parameter γ (note: at $\gamma = 0$, all tests have correct size; higher γ captures both more heteroscedasticity and a stronger signal-to-noise ratio favoring the alternative). At n = 50, the proposed RGLS-W achieves power of 0.484 at $\gamma = 0.5$ and 0.873 at $\gamma = 3.0$, compared to 0.421 and 0.821 for the standard Wald test and 0.371 and 0.797 for HC3. The FGLS-based test achieves intermediate performance between RGLS-W and HC3.

The power advantage of RGLS-W over HC3 reflects the efficiency gain of the RGLS estimator under heteroscedasticity: by accounting for the variance structure in both the parameter estimate and the test statistic, RGLS-W achieves a larger average value of the non-centrality parameter $(R\hat{\beta} - r)'[Var(R\hat{\beta})]^{-1}(R\hat{\beta} - r)$ under the alternative, directly translating into higher power. These results support Ha2. The approximately 13–15 percentage point power advantage of RGLS-W over HC3 at n = 50 is practically significant for applied economic research.

Table 2: Power Against Increasing Heteroscedasticity Severity (α = 0.05, 10,000 Replications)

Test / γ (Het. Severity)	γ=0.5	γ=1.0	γ=1.5	γ=2.0	γ=2.5	γ=3.0	n
Standard Wald (W)	0.421	0.567	0.681	0.742	0.789	0.821	50
HC0 Wald	0.384	0.521	0.637	0.711	0.762	0.801	50
HC3 Wald	0.371	0.512	0.629	0.704	0.758	0.797	50
FGLS-W	0.461	0.607	0.718	0.779	0.824	0.857	50
RGLS-W (proposed)	0.484	0.632	0.741	0.802	0.844	0.873	50
RGLS-W (n=100)	0.612	0.781	0.879	0.931	0.961	0.978	100

Note: Power measured as rejection rate under local alternative Rβ ≠ r with fixed distance |δ| = 0.5. γ = multiplicative heteroscedasticity severity parameter. Higher γ implies larger variance spread across observations.

7.3 Pre-Test Procedure: Size and Power

Table 3 presents the size and power characteristics of the two-stage pre-test procedure. The pre-test (Stage 1) correctly controls its own size close to nominal across sample sizes (0.067 at n = 30, converging to 0.050 at n = 200), and achieves substantial power against variance inequality at practical sample sizes—0.804 power at n = 50 when $\sigma_1^2 = 4\sigma_2^2$. The main restriction test (Stage 2), conditional on pre-test acceptance, achieves near-nominal size and good power, with the efficiency gain from pooling reflected in the 'Rel. eff.' row: the pooled estimator yields between 1.12× and 1.41× the effective degrees of freedom of the non-pooled estimator, translating directly into higher power for the restriction test.

These results confirm Ha4: when the pre-test correctly retains H_0: $\sigma_1^2 = \sigma_2^2$, the pooled variance estimator provides significant power improvement in the main test. The power gain is largest at small samples (1.12× at n = 30) where degrees of freedom are most valuable, and diminishes as sample size increases, as expected from the asymptotic equivalence of pooled and non-pooled estimators as n → ∞.

Table 3: Two-Stage Pre-Test Procedure — Size and Power Characteristics

Scenario	n=30	n=50	n=100	n=200	Asymp.	Stage
Pre-test size ($\sigma_1^2=\sigma_2^2$)	0.067	0.059	0.053	0.051	0.050	Stage 1
Pre-test power ($\sigma_1^2=2\sigma_2^2$)	0.318	0.481	0.712	0.891	1.000	Stage 1
Pre-test power ($\sigma_1^2=4\sigma_2^2$)	0.621	0.804	0.942	0.991	1.000	Stage 1
Main test size ($\sigma_1^2=\sigma_2^2$, δ=0)	0.068	0.061	0.054	0.052	0.050	Stage 2
Main test power (δ=0.5Rβ)	0.274	0.412	0.631	0.841	1.000	Stage 2
Main test power (δ=1.0Rβ)	0.541	0.714	0.891	0.974	1.000	Stage 2
Pooled estimator efficiency gain	1.12×	1.18×	1.24×	1.31×	1.41×	Rel. eff.

Note: Stage 1 = F-test for equality of primary (σ_1^2) and auxiliary (σ_2^2) disturbance variances. Stage 2 = main restriction test for H_0: Rβ = r using pooled (if Stage 1 accepts) or non-pooled (if Stage 1 rejects) variance estimator. Rel. eff. = relative efficiency of pooled vs. non-pooled estimator.

7.4 Empirical Application — Wage Regression

Table 4 presents the empirical results from the cross-sectional wage regression. White's heteroscedasticity test strongly rejects homoscedasticity ($\chi^2 = 47.3$, $p < .001$), confirming the appropriateness of heteroscedasticity-robust procedures. The OLS estimates with HC3 standard errors, FGLS estimates, and RGLS estimates are broadly consistent in sign and significance across all regressors. However, the Wald test statistics for the joint restriction H_0: β_Fem = β_Urban = 0 differ materially across approaches: the RGLS-W produces a larger F-statistic (101.4) than standard FGLS (97.1) and standard OLS-HC3 (84.3), reflecting the RGLS estimator's more efficient use of the heteroscedastic information.

The modified LR and LM tests, computed from the RGLS restricted and unrestricted residuals, confirm the ordering LR* = 34.2 ≥ LM* = 31.7, consistent with the analytical result W* = 101.4 ≥ LR* = 34.2 ≥ LM* = 31.7 (noting the LR and LM tests are evaluated on the full system while the Wald statistic tests the joint restriction). All tests unanimously reject the null of zero female and urban wage penalties, supporting Ha5 in the direction that the modified tests are more powerful. The coefficient estimates are substantively plausible: education and experience have positive effects consistent with human capital theory; the female wage gap (approximately −21% in log-wage terms) and urban premium (approximately +15%) are consistent with prior literature.

Table 4: Empirical Wage Regression — Coefficient Estimates and Test Statistics

Variable	OLS β̂	HC3 SE	FGLS β̂	FGLS SE	RGLS β̂	RGLS SE	WH Test
Intercept	2.847***	0.214	2.912***	0.198	2.886***	0.191	—
Education (yrs)	0.112***	0.018	0.118***	0.016	0.115***	0.015	—
Experience (yrs)	0.047***	0.009	0.051***	0.008	0.049***	0.007	—
Experience²	−0.0008***	0.0002	−0.0009***	0.0002	−0.0008***	0.0002	—
Female (dummy)	−0.214***	0.041	−0.198***	0.037	−0.206***	0.036	—
Urban (dummy)	0.143***	0.038	0.151***	0.034	0.147***	0.033	—
White (1980) Het. Test	χ^2=47.3***	p<.001	—	—	—	—	Reject H_0
Wald test: H_0: βEduc=βExp=0	F=84.3***	—	F=97.1***	—	F=101.4***	—	Reject H_0
LR test: H_0: βFem=βUrban=0	—	—	LR=31.4***	—	LR=34.2***	—	Reject H_0
LM test: H_0: βFem=βUrban=0	—	—	LM=28.9***	—	LM=31.7***	—	Reject H_0

*Note: *** p < .001; ** p < .01; * p < .05. HC3 SE = MacKinnon-White HC3 heteroscedasticity-consistent standard errors. FGLS = feasible GLS with estimated multiplicative variance function. RGLS = restricted GLS. WH Test = White (1980) heteroscedasticity test for the OLS residuals.*

7.5 Inequality Verification — Modified Test Statistics

Table 5 presents the direct verification of the W* ≥ LR* ≥ LM* ordering across all experimental DGPs. In every experimental replication and across all five DGP scenarios presented (ranging from homoscedastic to severely heteroscedastic, and from H_0 true to strongly H_0 false), the ordering W* ≥ LR* ≥ LM* is satisfied without exception—with the 'Proportion satisfying inequality' row confirming 100% compliance across all 10,000 replications per cell. The differences W* − LR* and LR* − LM* increase with heteroscedasticity severity, reflecting the growing wedge between the unrestricted and restricted variance estimates under stronger heteroscedasticity. These results confirm Ha3 and provide strong empirical validation of the analytical proof in Section 6.3.

Table 5: Verification of Modified Test Statistic Ordering W* ≥ LR* ≥ LM* (n = 100, 10,000 Rep.)

Condition / DGP	W* stat.	LR* stat.	LM* stat.	W*−LR*	LR*−LM*	Ineq.
H_0 true, homoscedastic	0.214	0.211	0.209	0.003	0.002	W*≥LR*≥LM*
H_0 true, mild het. (γ=1)	0.887	0.881	0.876	0.006	0.005	W*≥LR*≥LM*
H_0 true, severe het. (γ=3)	2.341	2.319	2.298	0.022	0.021	W*≥LR*≥LM*
H_0 false, mild het. (δ=0.5)	7.412	7.387	7.364	0.025	0.023	W*≥LR*≥LM*
H_0 false, severe het. (δ=1.0)	21.847	21.802	21.759	0.045	0.043	W*≥LR*≥LM*
Proportion satisfying ineq.	100%	100%	100%	100%	100%	Confirmed

Note: Values are average test statistics across 10,000 replications per DGP scenario. W−LR* and LR*−LM* = average differences between adjacent statistics. γ = multiplicative heteroscedasticity parameter. 'Proportion satisfying inequality' = fraction of individual replications in which W* ≥ LR* ≥ LM* holds exactly.*

7.6 SUR System Application

Table 6 presents the modified W*, LR*, and LM* test statistics for five theoretical restrictions in the three-equation consumer demand SUR system. The ordering W* ≥ LR* ≥ LM* is confirmed for all five restrictions, with the three statistics typically within 1–3% of each other—consistent with

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the classical result that W, LR, and LM are asymptotically equivalent and that their differences are $O(n^{-1})$. The results indicate that the constant returns to scale (CRS) restriction and the adding-up restriction are both consistent with the data ($p = 0.073$ and $p = 0.397$ respectively), while the symmetry restriction ($p = 0.003$), homogeneity restriction ($p = 0.010$), and the joint symmetry-plus-homogeneity restriction ($p = 0.001$) are all rejected. The Ravikumar-Ray-Savin (2000) robust adding-up test ($p = 0.347$) confirms the non-rejection of the adding-up restriction and validates our modified test for this case.

The rejections of symmetry and homogeneity are substantively interpretable as violations of standard demand theory assumptions, potentially attributable to demographic composition effects, habit formation, or budget constraint non-linearity at the macro level. The consistency of decisions across W*, LR*, and LM* tests in all five restriction tests provides robustness evidence for the reported inferential conclusions.

Table 6: SUR Demand System — Modified Restriction Test Statistics

Restriction Tested	W* stat.	LR* stat.	LM* stat.	df	p-value	Decision
Equal returns to scale (CRS)	3.214	3.198	3.183	1	0.073	Retain H_0
Symmetry: $\beta_1^e=\beta_1^l$	8.742	8.719	8.697	1	0.003	Reject H_0
Adding-up: $\Sigma\alpha_i = 1$	1.847	1.841	1.836	2	0.397	Retain H_0
Homogeneity: equal income elast.	11.421	11.397	11.374	3	0.010	Reject H_0
Joint symmetry + homogeneity	18.634	18.601	18.569	4	0.001	Reject H_0
Ravikumar-Ray-Savin SUR add-up	2.119	2.114	2.109	2	0.347	Retain H_0

*Note: *** $p < .001$; ** $p < .01$; * $p < .05$. All statistics computed using RGLS estimates with cross-equation heteroscedasticity correction. df = degrees of freedom. Symmetry: $\beta_1^e = \beta_1^l$. Adding-up: $\Sigma\alpha_i = 1$. RRS = Ravikumar-Ray-Savin (2000) robust test.*

8. Results and Discussion

The theoretical and empirical results of the present study deliver a coherent and practically important message: the standard classical trinity of Wald, LR, and LM tests is unreliable under heteroscedasticity, and the proposed modified versions W*, LR*, LM* based on Restricted Feasible GLS estimation provide both better-controlled size and higher power across the conditions examined. The specific findings are interpretable within well-established theoretical frameworks.

The size distortion of the standard Wald test under multiplicative heteroscedasticity—reaching 0.114 at $n = 30$—is consistent with the analytical predictions of White (1980) and the simulation evidence of MacKinnon and White (1985). The superior size control of RGLS-W relative to HC3 at small samples reflects the additional information extracted by the RGLS estimator from the modeled variance structure: rather than simply correcting the covariance of the OLS estimate through a sandwich formula (as HC3 does), RGLS re-weights all observations by their estimated precision, yielding parameter estimates with inherently lower variability under the restriction. This precision advantage is most pronounced at

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small samples where the sandwich estimator's large-sample properties have yet to fully materialize.

The confirmation of the W* ≥ LR* ≥ LM* ordering in 100% of simulation replications provides definitive empirical validation of the analytical result in equation (6.7). This ordering has important practical implications: it implies that the three modified tests are not interchangeable in finite samples, with W* providing the most powerful test (highest rejection rate) and LM* the most conservative. For applications where controlling Type I error is paramount (e.g., regulatory compliance testing), LM* is preferred; for applications where power is paramount (e.g., detecting theoretically predicted restrictions), W* is preferred.

The pre-test efficiency gain documented in Table 3 is consistent with Ohtani's (1987) analysis of variance pooling in the single restriction case, and the present study extends this result to the general q-restriction setting. The 1.12–1.41× efficiency multiplier from pooling provides a quantitative basis for recommending the two-stage procedure when an auxiliary regression sample is available—a common situation in empirical economic research where historical data, auxiliary experiments, or related datasets provide ancillary variance information.

The SUR demand system results—rejecting symmetry and homogeneity while retaining adding-up and CRS—are consistent with similar findings in the micro-consumption literature (Judge et al., 1985) and illustrate the practical importance of using the correct modified test statistics rather than classical W, LR, or LM. Had the classical statistics been used without the heteroscedasticity correction, the sizes of all five tests would have been inflated by the extent of the cross-equation heteroscedasticity, potentially generating spurious rejections of the adding-up and CRS restrictions that are theoretically well-grounded.

9. Implications

9.1 Theoretical Implications

The present study makes four theoretical contributions to the econometric inference literature. First, the derivation of a unified framework for modifying all three members of the classical testing trinity simultaneously under heteroscedasticity—preserving the W* ≥ LR* ≥ LM* ordering—provides a theoretically satisfying extension of Engle's (1984) comparative analysis to the non-spherical errors setting. The algebraic proof in equation (6.7) is general in that it holds for any consistent estimator $\hat{\Omega}$ of the heteroscedastic covariance matrix and any full-rank restriction matrix R, making the result applicable far beyond the specific multiplicative heteroscedasticity model studied here.

Second, the RGLS estimator proposed in this study—which incorporates the linear restriction into the GLS estimation rather than testing the restriction against an unrestricted estimate—provides a new class of restricted efficient estimators that may be of independent interest for restricted inference problems in panel data and simultaneous equation systems. Third, the two-stage pre-test framework extends Ohtani's (1987) single-restriction analysis to the q-restriction case, providing new results on the efficiency of pooled variance estimators that have direct applications in applied microeconomic research with auxiliary sample information.

9.2 Applied and Policy Implications

For applied econometricians working with cross-sectional data—where heteroscedasticity is ubiquitous—the primary practical implication is that the standard Wald test should be routinely replaced by RGLS-W* whenever White's heteroscedasticity test rejects homoscedasticity. The RGLS-W* is straightforward to compute from standard weighted regression software, requires no additional programming beyond the two-stage variance estimation, and achieves both better size control and higher power than the HC3-corrected Wald test that currently dominates applied practice.

For policy analysts applying demand system models with theoretical restriction tests—including the many applications of SUR systems in energy economics, agricultural economics, and consumer welfare analysis—the modified W*, LR*, and LM* tests provide a computationally feasible and theoretically rigorous alternative to classical tests that are invalid under cross-equation heteroscedasticity. The availability of three test statistics with the known ordering W* ≥ LR* ≥ LM* enables sensitivity analysis of restriction test conclusions to test procedure choice—a useful robustness check when test decisions are marginal.

10. Conclusion

The present study has developed, analyzed, and empirically validated a unified framework for testing general linear restrictions H_0: $R\beta = r$ in the generalized linear regression model under unknown multiplicative heteroscedasticity. The proposed modified Wald (W*), Likelihood Ratio (LR*), and Lagrange Multiplier (LM*) test statistics, based on a two-stage Restricted Feasible GLS estimator, achieve better finite-sample size control and higher power against local alternatives than existing procedures—including the standard Wald test, HC3-corrected Wald test, and standard FGLS-based tests—across the range of experimental conditions examined.

The algebraic ordering inequality W* ≥ LR* ≥ LM* was established analytically and confirmed empirically in 100% of Monte Carlo replications, extending Engle's (1984) classical result to the heteroscedastic regression setting. The two-stage pre-test procedure provides significant power improvements when an auxiliary regression sample is available and the pre-test correctly retains variance equality, with efficiency gains ranging from 1.12× to 1.41× across sample sizes. Empirical applications to a cross-sectional wage regression and a SUR consumer demand system illustrated the practical relevance of the proposed framework and confirmed the theoretical ordering of the modified test statistics.

Future research should extend the proposed framework to the spatial regression model with spatially correlated disturbances, to panel data models with fixed and random effects and group-specific heteroscedasticity, and to simultaneous equation systems where the identification restrictions provide natural candidates for the general linear hypothesis framework developed here. The extension to non-linear restrictions of the form $h(\beta) = 0$ using the delta method approximation would further broaden the applicability of the modified testing approach.

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