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Title: **ABOUT CHECKING IT TO THE EXTREMUM AT THE BREAKPOINTS OF THE FUNCTION**

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ABOUT CHECKING IT TO THE EXTREMUM AT THE BREAKPOINTS OF THE FUNCTION

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Abstract: In the current textbook, the subject of extremum testing of function is sufficiently broad and complete to describe the continuity of the function at the points of extremum testing, that is, the critical state of the function is not sufficiently covered. this article focuses on this aspect of the issue.
Keywords: function, interval, product, limit, breakpoints, minimum, maximum

Introduction

It is known that in the current textbook, in the textbooks on the examination of the function of the extremum, the state of continuity of the function at the critical points, ie at the critical points, is sufficiently broadly and completely covered, but the cases of interruption of the function at the extremum test points are not sufficiently covered. In this article, we aim to focus on this aspect of the issue.

First of all, let's take a look at the basic information given in the current textbook on the issue of the extremum of a function.

The function $y = f(x)$ is defined in the interval (a,b), let $x_0 \in (a,b)$ be.

Definition 1. If there is a circumference $(x_0 - S,\ x_0 + S)$ of the point x_0, and the inequality $f(x) \le f(x_0)$ $(f(x) \ge f(x_0))$ holds for any x obtained from this circumference, then the point x_0 is called the maximum (minimum) point of the function $f(x)$, and the maximum (minimum) of the function $f(x_0)$.

Also, the maximum and minimum points of a function are called the extremum points of the function, and the maximum and minimum values are called the extremums of the function.

It is known that the extremum points of a function $f(x)$ are searched among the following points:

1) points satisfying the equation $f'(x) = 0$, such points are called stationary points of the function.

2) points where the product $f'(x)$of the function does not exist (at such points the function is continuous or intermittent).

Typically, such points are called critical points of the function.

For points where the function does not have stationary points and derivatives, but at which point the function is continuous, the extremum problem is solved by the following theorem:

Theorem 1. Let the function $f(x)$ be continuous at point x_0 and let x_0 be the critical point of the function.

a) If at any $(x_0 - S,\ x_0)$ $f`(x) > 0, (x_0, x_0 + S)$ the inequalities $f'(x) < 0$ are valid, ie the product $f'(x)$ changes its sign from "+" to "-" when passing through the point x_0, then the function $f(x)$ has a maximum at the point x.

b) If the inequality $f`(x) > 0$ at $(x_0 - S,\ x_0)$ and $f`(x) < 0$, $(x_0,\ x_0 + S)$ is valid, that is, if the product $f'(x)$ changes its sign from "-" to "+" when passing through the point x_0, then the function $f(x)$ has a minimum at the point x_0.

c) If the product $f`(x)$ does not change its sign when passing through point x_0, then the

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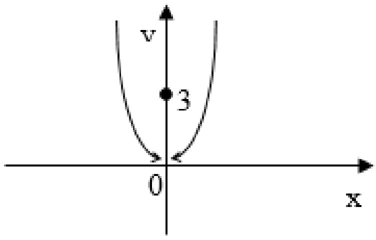
function $f(x)$ does not have an extremum at point x_0. [1]

However, Theorem 1 cannot be used if the critical point of the function has no derivative of the function and has a function interruption at such a point.

For example,

1-example .

$$f(x)=\begin{cases} x^2 & агар \quad x\neq 0 \\ 3 & агар \quad x=0 \end{cases}$$



the product of the function at the point $x=0$ does not exist, i.e. the point $x=0$ is the critical point of the function and the function has a break at that point. However, when the product $f'(x)$ changes its sign from "-" to "+" when passing through the point $x=0$, the function $f(x)$ has a maximum of $f(x)=3$, not a minimum at the point $x=0$.

This is because the function at point $x_0=0$ is not continuous.

The problem of the extremum at the critical points of the function with a break (where the product does not exist and has a break at this point) can be solved using the left and right limits of the function at this point.

First of all, let's remember the information about the breakpoints of a function and their types.

Definition 2. If the $\lim_{x\to a} f(x) = f(a)$ equation does not hold at point $x=a$, then point $x=a$ is called the breakpoint of the function $f(x)$.

Definition 3. If $f(a-0)=f(a+0)=b$ has a finite limit and $f(a)\neq b$ (i.e. $\lim_{x\to a} f(x)=b\neq f(a)$,), then the function at point $x=a$ is said to have an interrupt that can be eliminated.

Definition 4. If $f(a-0)$ and $f(a+0)$ has limits, is finite, and $f(a-0)\neq f(a+0)$, then the function at point $x=a$ is said to have a first-round interrupt.

Definition 5. If $f(a-0)$ ва $f(a+0)$ does not have at least one of the one-sided limits, or at least one of them is infinite, then the function at point $x=a$ is said to have a second round interrupt. [2]

We study the problem of extremum at critical points, which is the breaking point of the function, separately for the types of interruptions.

Theorem 2. The critical point of a function $f(x)$ is its breakpoint:

a) If $\lim_{x\to a} f(x)=b<f(a)$, then the function has a maximum at point $x=a$.

b) If $\lim_{x\to a} f(x) = b > f(a)$, then the function has a minimum at the point $x=a$.

Proof: a) Let us prove the case. It is known that if $\lim_{x\to a} f(x)=b$ is present and is finite $b<q$ (q-finite number), then $f(x)<q$ will be $f(x)<q$ for $x(x\neq a)$s obtained from a sufficiently small circumference of point a. (T. Azlarov, H. Mansurov. "Mathematical analysis". Part 1. 1994. p. 136). [3].

Using this property of a function with a finite limit, it follows that the inequality $f(x)<f(a)$ is appropriate for $x(x\neq a)$ around a sufficiently small circumference of the point $x=a$. That is, at point $x=a$, the function has a maximum.

b) it is still proved in the same way.

2-мисол. $f(x)=\begin{cases} x^2-2x+1 & агар \ x\neq 1 \\ -3 & агар \ x=1 \end{cases}$

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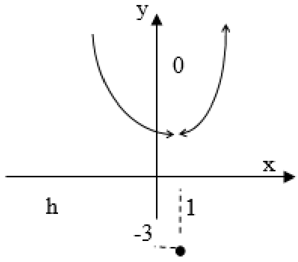


function has minimum at point $x=1$,

because $\lim_{x\to 1} f(x)=0>f(1)=-3$.

3-теорема. The critical point of a function is its first round breakpoint:

a) $\begin{cases} f(a-0)\le f(a) \\ f(a+0)<f(a) \end{cases}$ or $\begin{cases} f(a-0)<f(a) \\ f(a+0)\le f(a) \end{cases}$, then function has maximum at point $x=a$.

в) $\begin{cases} f(a-0)\ge f(a) \\ f(a+0)>f(a) \end{cases}$ or $\begin{cases} f(a-0)>f(a) \\ f(a+0)\ge f(a) \end{cases}$, then function has minimum at point $x=a$

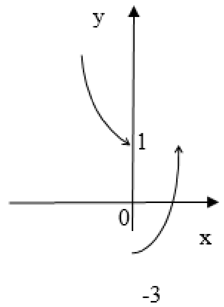
Proof. Theorem 2 is proved as a proof.

3-example.

$$f(x)=\begin{cases} x^2+1 & агар \quad x<0 \\ x^2-3 & агар \quad x\ge 0 \end{cases}$$

then function has minimum at point $x=0$,

because $f(-0)=1$, $f(+0)=-3$ and $f(0)=-3$.

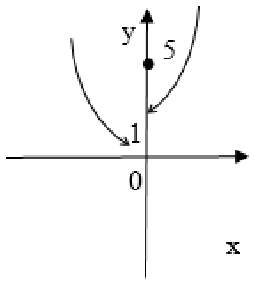


4-exapmle.

$$f(x)=\begin{cases} x^2 & arap \quad x<0 \\ 5 & arap \quad x=0 \\ x^2+1 & arap \quad x>0 \end{cases}$$

then function has maximum at point $x=0$,

because $f(-0)=0$, $f(+0)=1$, $f(0)=5$.



The critical point of the function is its study with a second round breakpoint

for the first round breakpoint all possible views

According to the table:

Table

The limit of the function at points x= a	$f(a-o)$ Left limit	$f(a+o)$ Right limit	Shape
There is not limit	A - limited	$+\infty$	1
	$+\infty$	A - limited	2
	A - limited	$-\infty$	3
	$-\infty$	A - limited	4
	A - limited	Not exist	5
	Not exist	A - limited	6
	$+\infty$	Not exist	7
	Not exist	$+\infty$	8
	$-\infty$	Not exist	9
	Not exist	$-\infty$	10
	Not exist	Not exist	11
	$+\infty$	$-\infty$	12
	$-\infty$	$+\infty$	13
$\lim_{x\to a} f(x)=\infty$	$+\infty$	$+\infty$	14
	$-\infty$	$-\infty$	15

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If at the critical point $x = a$ the function has the second type of breakpoints in 1,2,3,4,14 or 15 views, the problem of extremum at these points can be solved by the following theorem.

Theorem 4. If the critical point of a function $f(x)$ is one of its second-round breakpoints in 1,2,3,4,14 or 15 views,

a) $\begin{cases} f(a-0) \le f(a) \\ f(a+0) < f(a) \end{cases}$ or

$\begin{cases} f(a-0) < f(a) \\ f(a+0) \le f(a) \end{cases}$, then function has maximum at point $x = a$.

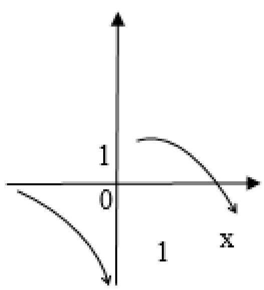
в) $\begin{cases} f(a-0) \ge f(a) \\ f(a+0) > f(a) \end{cases}$ or

$\begin{cases} f(a-0) > f(a) \\ f(a+0) \ge f(a) \end{cases}$, then function has minimum at point $x = a$

The proof of this theorem is also proved as Theorem 2.

5-example.

$$f(x) = \begin{cases} \dfrac{1}{x} & агар \quad x < 0 \\ -x^2 + 1 & агар \quad x \ge 0 \end{cases}$$



According to the theorem 4, function has maximum at point $x = 0$,

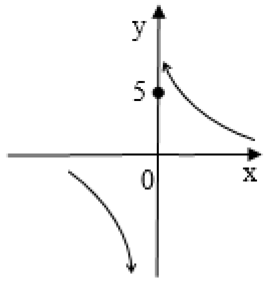
because $f(-0) = -\infty$ $f(+0) = 1$ or $f(0) = 1$.

Examples can also be given of all the other views given in Theorem 4.

If $x = a$ has a second round break point of view 12 or 13 at the critical point, the function will not have an extremum at such points.

6-exapmle.

$$f(x) = \begin{cases} \dfrac{1}{x} & агар \quad x \ne 0 \\ 5 & агар \quad x = 0 \end{cases}$$



$x = 0$ point is the critical point for the function

(i.e. there is no product at point $x = 0$)

and $x = 0$ is the second of the point functions

round breakpoint. Also, the function has no extremum at point $x = 0$.

There is no general rule for the issue of extremum at critical points with all other remaining 5,6,7,8,9,10 or 11 second-round interruptions. In such cases, the issue of extremum is considered separately depending on the assignment of functions.

7-example.

$$f(x) = \begin{cases} \dfrac{1}{x} & агар \quad x < 0 \\ 5 & агар \quad x = 0 \\ \sin\left(\dfrac{1}{x}\right) & агар \quad x > 0 \end{cases}$$

function has maximum at point $x = 0$,

because $f(-0) = -\infty$, $f(+0)$ -not exist,

but $\left|\sin\left(\dfrac{1}{x}\right)\right| \le 1$ ва $f(0) = 5$.

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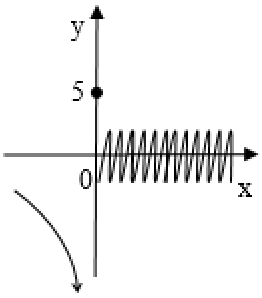

In conclusion, we believe that even in cases where the critical point of the function is interrupted, the issue of extremum is given in the textbooks based on the above facts and considerations, which leads to an increase in students' knowledge of the subject.

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