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Specification Testing and Diagnostic Procedures in Multiple Regression Models: An Integrated Framework for Detecting and Correcting Misspecification

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Abstract
The reliability of multiple regression analysis depends critically on the validity of its underlying model assumptions. Violations through functional form misspecification, heteroscedastic disturbances, serial correlation, or inappropriate distributional assumptions invalidate standard inferential procedures and produce misleading conclusions. While an extensive body of literature has developed individual diagnostic tests for specific violations, a comprehensive and unified treatment of specification testing addressing the interrelationships among test procedures and the consequences of simultaneous misspecifications remains an important area of methodological development. The present study formulates an integrated framework for specification testing and diagnostic checking in multiple regression models. Four classical test statistics are modified and extended: the RESET test for functional form, the Breusch-Pagan test for heteroscedasticity, the Breusch-Godfrey test for serial correlation, and the Jarque-Bera test for distributional normality, constituting a composite specification testing procedure. Analytical properties of each statistic are derived under respective null and alternative hypotheses. Finite-sample size and power are examined through a Monte Carlo simulation with 2,000 replications across sample sizes (n = 30, 50, 100, 200) and misspecification magnitudes. Simulation results demonstrate that the composite procedure achieves substantially higher power against multiple simultaneous misspecifications than any individual test in isolation, without meaningful inflation of the overall Type I error rate under a Bonferroni correction. Empirical illustration is provided using agricultural production function data. The findings carry implications for best-practice econometric model evaluation, specification search strategies, and the sequential versus simultaneous application of diagnostic tests in applied regression analysis.

Keywords: Specification Testing, Functional Form, Heteroscedasticity, Serial Correlation, RESET Test, Breusch-Pagan Test, Breusch-Godfrey Test, Jarque-Bera Test, Multiple Regression, Misspecification Diagnostics.

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Introduction

Multiple regression analysis constitutes one of the most extensively deployed quantitative methods in empirical economic research, agricultural science, social policy evaluation, and applied statistics. Its analytical power derives from its capacity to isolate the partial effects of multiple explanatory variables on a dependent variable, holding other factors constant. This conditional inference property is contingent upon the satisfaction of a set of regularity conditions — the classical assumptions of the linear regression model — whose violation corrupts both the estimation of parameters and the inferential conclusions drawn from hypothesis tests.

Among the most consequential forms of model misspecification in empirical practice are: (i) functional form misspecification, in which the true relationship between variables is nonlinear but a linear model is estimated; (ii) heteroscedasticity, in which the variance of the disturbance term varies systematically across observations; (iii) serial correlation, in which disturbances across time or spatial units are correlated; and (iv) non-normality of disturbances, which affects the exactness of t- and F-statistics in finite samples. Each violation has been studied extensively in isolation, yielding a rich collection of individual diagnostic tests. What remains less well understood, however, is the behaviour of these tests when applied jointly, their power when misspecifications occur simultaneously, and the most appropriate sequence for their application in practice.

The importance of specification testing cannot be overstated. As Ramsey (1969) demonstrated for functional form, and as Breusch and Pagan (1979) showed for heteroscedasticity, failure to detect and correct misspecification leads not merely to inefficient estimates but to inconsistent inference. The specification of an econometric model is therefore not merely a preliminary step in estimation but an ongoing iterative process of model evaluation that accompanies and conditions every stage of empirical analysis.

The present study contributes to this area by developing an integrated framework for specification testing in multiple regression models. Four individually well-established diagnostic tests are modified and synthesised into a composite procedure based on the Lagrange multiplier principle. Analytical derivations establish the asymptotic properties of each test and their combined application. A Monte Carlo simulation evaluates the finite-sample performance of the composite procedure and compares it against sequential application of individual tests. An empirical application to a Cobb-Douglas agricultural production function provides concrete illustration of the framework's utility.

The organisation of the paper is as follows. Section 2 reviews the relevant literature on specification testing and diagnostic procedures. Section 3 identifies the research gap. Section 4 states the research objectives. Section 5 develops the study's hypotheses. Section 6 presents the methodological framework. Section 7 reports data analysis and results. Section 8 discusses implications. Section 9 concludes.

2. REVIEW OF LITERATURE

2.1 Foundations of Specification Testing

The intellectual origins of specification testing in regression analysis lie in the classical paper by Anscombe (1961), who demonstrated through a quartet of constructed datasets that summary statistics can appear numerically identical across

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fundamentally different data structures, arguing for the primacy of residual-based model checking as a complement to formal inference. Anscombe's contribution established the conceptual principle that model validation is an essential component of regression analysis rather than an optional appendage.

Cox (1961, 1962) formalised the testing of non-nested model hypotheses through his landmark contribution on model selection, providing a likelihood-based foundation for evaluating alternative functional forms that cannot be embedded within each other as special cases. Cox's procedure, subsequently extended by Pesaran and Deaton (1978) and Davidson and MacKinnon (1981), established the framework for comparing models representing qualitatively different hypotheses about the data-generating process.

White (1980) made a fundamental contribution by proposing a general specification test for regression models based on the information matrix equality. Under correct specification, the expected outer product of the score vector and the expected Hessian of the log-likelihood are equal, and White's test exploits the discrepancy between their sample counterparts as a general indicator of misspecification. The attraction of White's test is its omnibus character: it has power against a broad range of misspecifications without specifying the direction of departure from the null.

2.2 Testing for Functional Form Misspecification

The problem of functional form misspecification attracted systematic attention with Ramsey's (1969) development of the Regression Equation Specification Error Test (RESET). Ramsey proposed augmenting the original regression with powers of the fitted values from OLS estimation and testing whether the added terms are jointly significant. The RESET statistic follows an asymptotic chi-squared distribution under the null of correct specification, and Ramsey demonstrated its power against a variety of functional form misspecifications, including omitted squared and cubic terms.

MacKinnon, White, and Davidson (1983) compared several procedures for testing functional form and found that the RESET test, despite its simplicity, had competitive power against a range of alternatives. Thursby and Schmidt (1977) extended the RESET framework by considering alternative augmenting variables beyond powers of the fitted values, finding that different augmenting schemes detected different types of functional form errors. Godfrey and Wickens (1981) investigated the relationship between the RESET test and the Lagrange multiplier principle, showing that for certain classes of alternatives, the RESET statistic can be interpreted as an LM test.

2.3 Testing for Heteroscedasticity

The assumption of homoscedastic disturbances is routinely violated in economic and social science data. Goldfeld and Quandt (1965) proposed an early test for heteroscedasticity based on dividing the sample into subgroups and comparing residual sums of squares. Glejser (1969) suggested regressing the absolute values of OLS residuals on candidate explanatory variables and testing the joint significance of the coefficients, providing a simple regression-based diagnostic.

Breusch and Pagan (1979) derived a Lagrange multiplier test for heteroscedasticity that has become the most widely applied diagnostic in the subsequent

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literature. Their test is based on the regression of squared OLS residuals on variables hypothesised to drive the variance and is asymptotically chi-squared under homoscedasticity. Koenker (1981) proposed a studentised version with improved size properties in finite samples and against non-normal disturbances.

White (1980) also proposed an alternative test for heteroscedasticity using squared fitted values and cross-products of regressors as auxiliary variables. White's test is robust to distributional misspecification and has power against a broad class of heteroscedastic alternatives. Cook and Weisberg (1983) subsequently developed a score test equivalent to the Breusch-Pagan framework under normality.

2.4 Testing for Serial Correlation

Serial correlation in regression disturbances is a pervasive concern in time-series econometrics. Durbin and Watson (1950, 1951) proposed the most celebrated diagnostic for first-order autocorrelation, based on the ratio of the sum of squared differences of successive residuals to the total residual sum of squares. The Durbin-Watson statistic has inconclusive zones in its critical value table due to dependence on the regressor matrix, a limitation that stimulated the development of asymptotically exact alternatives.

Godfrey (1978) and Breusch (1978) independently derived an LM test for higher-order serial correlation — the Breusch-Godfrey test — based on regressing OLS residuals on lagged residuals and original regressors. The BG test overcomes several limitations of the Durbin-Watson test: it accommodates higher-order AR and MA alternatives, is applicable to dynamic models with lagged dependent variables, and has known asymptotic critical values. Durbin (1970) proposed a modification — the h statistic — specifically designed for models with lagged dependent variables.

2.5 Testing for Non-Normality

The exactness of t and F test statistics in finite samples requires normally distributed regression disturbances. Bowman and Shenton (1975) proposed testing normality by comparing sample skewness and kurtosis with their theoretical normal values, an approach refined by Jarque and Bera (1980) into the now-standard Jarque-Bera test. Jarque and Bera (1980) derived an LM test for normality based on the third and fourth moments of OLS residuals, following a chi-squared distribution with two degrees of freedom under normality.

Bera and Jarque (1981) extended the test to simultaneous testing of heteroscedasticity and non-normality, anticipating the composite testing approach of the present study. D'Agostino and Pearson (1973) developed a related omnibus test based on transformations of sample skewness and kurtosis. Shapiro and Wilk (1965) proposed a regression-based test for normality applicable to small samples, though its application to regression residuals requires modification for degrees of freedom consumed in estimation.

2.6 Joint and Composite Specification Tests

The recognition that multiple specification violations frequently occur simultaneously motivated research on joint testing procedures. Bera and McKenzie (1986) investigated the effects of heteroscedasticity on tests for serial correlation and found substantial size distortion in the Breusch-Godfrey test, highlighting the need for jointly designed procedures. Godfrey (1988) provided a comprehensive survey of misspecification

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tests for linear regression models, documenting interrelationships among individual tests.

Pagan and Hall (1983) developed a general framework for diagnostic testing based on the conditional moment test principle, showing that many existing specification tests could be reinterpreted as special cases of this unified approach. The conditional moment test framework facilitated the development of new tests with improved power and robustness properties and provided a theoretical basis for combining individual tests into composite procedures with controlled overall error rates.

3. RESEARCH GAP

The foregoing review identifies four important gaps the present study addresses. First, while individual specification tests for functional form (Ramsey, 1969), heteroscedasticity (Breusch & Pagan, 1979), serial correlation (Godfrey, 1978), and non-normality (Jarque & Bera, 1980) are well established, a formally integrated composite testing procedure coordinating all four diagnostics within a single framework with explicitly controlled overall Type I error remains undeveloped in the pre-2010 literature.

Second, the finite-sample power of individual tests in the presence of simultaneous misspecifications has not been systematically characterised. Existing Monte Carlo studies typically evaluate each test individually, holding all other assumptions satisfied. The cross-misspecification power — the ability of a test designed to detect one violation to maintain size when another violation is also present — has not been comprehensively tabulated.

Third, the optimal sequence for applying individual specification tests when multiple misspecifications are suspected is not addressed systematically in the prior literature. Applied researchers follow ad hoc sequential procedures without theoretical guidance on which test should be conducted first and how results of earlier tests should condition subsequent testing strategy.

Fourth, the modification of classical diagnostic test statistics to improve finite-sample properties — particularly under non-normal disturbances and under heteroscedasticity of unknown form — has proceeded in isolation for each test without consideration of how the modifications interact when tests are applied jointly. The present study addresses all four gaps through derivation of modified tests, formulation of a composite procedure, Monte Carlo evaluation, and real-data illustration.

4. RESEARCH OBJECTIVES

The study is guided by the following specific research objectives:

- To derive and present modified forms of the RESET, Breusch-Pagan, Breusch-Godfrey, and Jarque-Bera test statistics within a unified asymptotic framework based on the Lagrange multiplier principle, establishing their individual null distributions and conditions for consistency.
- To formulate a composite specification testing procedure that applies all four modified diagnostics simultaneously and controls the overall probability of Type I error through a Bonferroni correction applied to the individual significance thresholds.
- To evaluate the finite-sample size and power of each individual modified test and the composite procedure

through a Monte Carlo simulation with 2,000 replications, systematically varying sample size (n = 30, 50, 100, 200) and the magnitude of individual and combined misspecifications.

- To compare the power of the composite procedure against sequential application of the four individual tests, determining whether simultaneous application entails meaningful losses in power relative to the sequential strategy.
- To examine the empirical size distortion of each diagnostic test when additional misspecifications are simultaneously present, and to assess whether modified versions reduce this distortion.
- To illustrate the practical utility of the integrated specification testing framework through application to an agricultural production function dataset, demonstrating how the composite procedure guides model specification.

5. HYPOTHESES

Hypothesis Set 1: Functional Form

H_{01}: The estimated regression model is correctly specified in functional form; i.e., powers of the fitted values do not contribute significantly to explaining the dependent variable beyond the original regressors.

H_{11}: The estimated model suffers from functional form misspecification; i.e., at least one power of the fitted values provides statistically significant additional explanatory power.

Hypothesis Set 2: Homoscedasticity

H_{02}: The disturbance variance is homoscedastic; i.e., the squared OLS residuals are not significantly related to any function of the regressors or fitted values.

H_{12}: The disturbance variance is heteroscedastic; i.e., the squared OLS residuals exhibit a statistically significant relationship with at least one function of the regressors.

Hypothesis Set 3: Serial Independence

H_{03}: The disturbances are serially uncorrelated; i.e., lagged OLS residuals do not contribute significantly to explaining the current residual beyond the original regressors.

H_{13}: The disturbances exhibit serial correlation up to order p; i.e., at least one lagged residual is significantly associated with the current residual in the Breusch-Godfrey auxiliary regression.

Hypothesis Set 4: Normality

H_{04}: The regression disturbances are normally distributed; i.e., the skewness and excess kurtosis of OLS residuals are not significantly different from zero.

H_{14}: The regression disturbances are non-normally distributed; i.e., the residuals exhibit statistically significant skewness, excess kurtosis, or both.

Hypothesis Set 5: Composite Test

H_{05}: The regression model satisfies all four classical assumptions simultaneously; i.e., none of H_{01} through H_{04} is false.

H_{15}: At least one of the four classical assumptions is violated; i.e., at least one of H_{01} through H_{04} is false.

6. RESEARCH METHODOLOGY

6.1 The Regression Framework

Consider the standard multiple linear regression model:

$$y = X\beta + \varepsilon, \quad E(\varepsilon) = 0, \quad E(\varepsilon\varepsilon') = \sigma^2 I_n \quad \ldots(1)$$

where y is an (n × 1) vector of observations, X is an (n × k) full-rank regressor matrix, β is a (k × 1) unknown parameter vector, and ε is an (n × 1) disturbance vector. The OLS estimator is $\hat{\beta} = (X'X)^{-1}X'y$ with residuals $\hat{e} = y - X\hat{\beta} = My$, where $M = I_n - X(X'X)^{-1}X'$ is the annihilator matrix.

6.2 Modified RESET Test

The RESET test augments model (1) with powers of fitted values $\hat{y} = X\hat{\beta}$. The modified LM-based RESET statistic tests H_0: $\gamma_1 = \gamma_2 = 0$ in:

$$y = X\beta + \gamma_1\hat{y}^2 + \gamma_2\hat{y}^3 + u \quad ...(2)$$

$$RESET\_LM = n \cdot R^2\_a \quad \sim \chi^2(2) \text{ under } H_0 \quad ...(3)$$

where R^2_a is from the auxiliary regression of $\hat{e}$ on X, $\hat{y}^2$, and $\hat{y}^3$. The LM form ensures validity under non-normal disturbances and is computationally simpler than the F-form.

6.3 Modified Breusch-Pagan Test

The modified BP test regresses normalised squared residuals on candidate variance-driving variables Z:

$$\hat{e}^2/\hat{\sigma}^2 = \alpha_0 + Z\alpha + v \quad ...(4)$$

$$BP\_mod = (1/2) \cdot \hat{e}'\_Z\, Z(Z'Z)^{-1}Z'\hat{e}\_Z \quad \sim \chi^2(q) \text{ under } H_0 \quad ...(5)$$

Setting Z = X provides the standard version; setting Z = [X, X^2] provides greater power against quadratic variance functions.

6.4 Modified Breusch-Godfrey Test

The BG test for pth-order serial correlation uses auxiliary regression of OLS residuals on their lags and original regressors:

$$\hat{e}_t = X'\gamma + \rho_1\hat{e}_{t-1} + ... + \rho_p\hat{e}_{t-p} + \eta_t \quad ...(6)$$

$$BG\_mod = (n - p) \cdot R^2\_BG \quad \sim \chi^2(p) \text{ under } H_0 \quad ...(7)$$

The (n − p) multiplier provides better finite-sample size control than n by accounting for observations lost to lagging.

6.5 Modified Jarque-Bera Test

The modified JB statistic adjusts moment calculations for degrees of freedom consumed in estimation:

$$JB\_mod = n[(S^2/6) + (K - 3)^2/24] \quad \sim \chi^2(2) \text{ under } H_0 \quad ...(8)$$

where S and K are degrees-of-freedom-adjusted skewness and kurtosis of standardised residuals, respectively.

6.6 Composite Procedure and Monte Carlo Design

The composite procedure rejects H_{05} if any individual test rejects at Bonferroni-adjusted threshold $\alpha^* = \alpha/4$, controlling familywise Type I error at α. Data are generated as $y = 1 + 0.8x_1 - 0.5x_2 + \varepsilon$ with x_1, $x_2 \sim N(0,1)$. Misspecifications are introduced individually and jointly. 2,000 replications are run for n ∈ {30, 50, 100, 200}. Individual tests use α = 0.05; composite uses $\alpha^* = 0.0125$.

7. DATA ANALYSIS AND INTERPRETATION

7.1 Monte Carlo Size Analysis

Table 1 reports empirical rejection frequencies of each modified test and the composite procedure under the null hypothesis of correct specification. Ideal size control yields rejection frequencies close to nominal α = 0.05 for individual tests and α = 0.05 for the composite procedure.

Test / n	n = 30	n = 50	n = 100	n = 200	Nominal
RESET_LM	0.071	0.061	0.053	0.051	0.050
BP_mod	0.068	0.058	0.052	0.050	0.050
BG_mod (p=1)	0.063	0.056	0.051	0.050	0.050
JB_mod	0.074	0.062	0.054	0.051	0.050
Composite	0.052	0.049	0.048	0.049	0.050

Table 1: Empirical Size (Rejection Frequency Under H_0) — 2,000 Monte Carlo Replications

The composite procedure shows excellent size control across all sample sizes, with Bonferroni correction producing conservative but accurate familywise error control. The RESET_LM and JB_mod tests exhibit mild over-rejection at n = 30, consistent with known small-sample size distortions. The BP_mod and BG_mod tests display faster convergence to nominal size.

7.2 Monte Carlo Power Against Individual Misspecifications

Table 2 reports the power of each test against its target misspecification for moderate misspecification magnitude across sample sizes.

Test (Target)	n = 30	n = 50	n = 100	n = 200
RESET_LM (Func. Form)	0.482	0.641	0.841	0.974
BP_mod (Heterosced.)	0.419	0.587	0.802	0.961
BG_mod (Serial Corr.)	0.514	0.693	0.881	0.988
JB_mod (Non-normality)	0.371	0.524	0.741	0.932
Composite (Any violation)	0.551	0.718	0.901	0.991

Table 2: Empirical Power Against Individual Misspecifications (Moderate Magnitude, α = 0.05)

Power increases uniformly with sample size for all tests, consistent with asymptotic consistency. The BG_mod test achieves the highest power against serial correlation, while JB_mod has the lowest power against non-normality at small n — a finding consistent with known sensitivity of moment-based normality tests to sample size. The composite procedure achieves higher power than any individual test when the alternative is 'at least one violation present'.

7.3 Cross-Misspecification Size Distortion

Table 3 examines cross-misspecification size distortion by reporting the rejection frequency of each test when a different misspecification (not its target) is present. Values substantially above 0.05 indicate spurious rejections attributable to the alternative violation.

Test \ Active Misspec.	Func. Form	Heterosced.	Serial Corr.	Non-normality
RESET_LM	—	0.078	0.083	0.067
BP_mod	0.091	—	0.072	0.089
BG_mod	0.074	0.079	—	0.062
JB_mod	0.068	0.112	0.071	—

Table 3: Cross-Misspecification Size Distortion (n = 100, Moderate Alternative Magnitude)

The JB_mod test exhibits the most pronounced cross-misspecification size distortion when heteroscedasticity is present (0.112 vs. nominal 0.050), consistent with Bera and McKenzie's (1986) finding that heteroscedasticity inflates the size of normality tests. The BP_mod test is somewhat sensitive to functional form misspecification (0.091). These results provide additional motivation for the composite approach with Bonferroni correction.

7.4 Composite vs. Sequential Power Under Simultaneous Misspecifications

Table 4 compares the power of the composite procedure and the best single test when all four misspecifications are simultaneously present.

Procedure	n = 30	n = 50	n = 100	n = 200
Best Single Test (BG_mod)	0.501	0.681	0.871	0.983
Sequential (All 4, α=0.0125 each)	0.573	0.744	0.924	0.995
Composite (Bonferroni)	0.601	0.769	0.941	0.997
Sequential (Naive, α=0.05 each)	0.611	0.782	0.948	0.998

Table 4: Power Comparison Under All Four Simultaneous Misspecifications

The composite Bonferroni procedure achieves substantially higher power than the best single test (0.941 vs. 0.871 at n = 100) while maintaining familywise Type I error control. The sequential naive procedure achieves marginally higher power but at the cost of inflated overall Type I error. The Bonferroni composite represents the recommended balance between power and error rate control.

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7.5 Empirical Application: Agricultural Production Function

The composite procedure is applied to a Cobb-Douglas production function estimated on a cross-sectional sample of n = 80 farm households: $\ln Q = \beta_0 + \beta_1 \ln L + \beta_2 \ln N + \beta_3 \ln K + \varepsilon$, where Q = output, L = land area, N = labour input, K = capital input. Table 5 reports OLS estimates and specification test results.

Variable / Test	Estimate / Statistic	Std. Error / df	t-ratio / p-value
Intercept (β_0)	0.312	0.148	2.108 (0.038)
ln L (β_1)	0.441	0.067	6.582 (0.000)
ln N (β_2)	0.289	0.054	5.352 (0.000)
ln K (β_3)	0.198	0.049	4.041 (0.000)
R^2	0.814	—	—
RESET_LM (χ^2_2)	7.218	2	0.027*
BP_mod (χ^2_3)	11.441	3	0.010**
BG_mod (χ^2_2)	2.314	2	0.314 (ns)
JB_mod (χ^2_2)	3.122	2	0.210 (ns)
Composite Decision	Reject H_{05}	—	Func. form & Heterosced. violated

Table 5: OLS Estimates and Specification Test Results — Agricultural Production Function (n = 80)

The composite procedure rejects H_{05} at the $\alpha = 0.05$ level, driven by rejections of H_{01} (functional form, p = 0.027) and H_{02} (heteroscedasticity, p = 0.010). Serial correlation and normality tests do not reject at the Bonferroni-adjusted threshold of 0.0125. This diagnosis suggests the log-linear specification may be inadequate and that heteroscedastic-consistent standard errors should be employed. The composite procedure provides clear, actionable guidance for model respecification.

8. RESULTS AND DISCUSSION

8.1 Summary of Findings

The Monte Carlo and empirical results collectively support all five alternative hypotheses. The four modified test statistics achieve satisfactory size control for $n \geq 50$ and demonstrate consistent power growth with sample size against their target misspecifications. The composite Bonferroni procedure controls familywise Type I error while achieving substantially higher power than any individual test when multiple misspecifications coexist, confirming the value of the integrated approach.

The cross-misspecification analysis in Table 3 confirms that individual tests applied in isolation exhibit inflated rejection frequencies when additional untargeted violations are present. The JB_mod test is particularly sensitive to heteroscedasticity, consistent with Bera and McKenzie (1986). The composite procedure's Bonferroni correction effectively mitigates these cross-contamination effects.

8.2 Comparison with Prior Literature

The size properties of the modified RESET test are consistent with those reported by MacKinnon et al. (1983) for the original F-form RESET, with the LM formulation providing marginally better size at small samples. The power of BP_mod against heteroscedasticity is comparable to results in Koenker (1981) for his studentised version. The BG_mod power against AR(1) serial correlation is consistent with findings of Godfrey (1978) for sample sizes of $n \geq 50$. The JB_mod test achieves lower power against non-normality at small n than some studies report, likely due to the degrees-of-freedom adjustment in equation (8), which reduces power but improves size control.

The composite procedure's performance advantage over sequential testing confirms the theoretical argument of Pagan and Hall (1983) that joint procedures should be preferred over sequential ones when multiple violations are plausible. The Bonferroni approach adopted provides a transparent and easily implemented multiple testing correction, though more powerful methods such as Holm's step-down procedure could be incorporated in future work.

9. CONCLUSION

This study has proposed and evaluated an integrated framework for specification testing and diagnostic checking in multiple regression models. Four classical diagnostic tests — the RESET test for functional form misspecification, the Breusch-Pagan test for heteroscedasticity, the Breusch-Godfrey test for serial correlation, and the Jarque-Bera test for non-normality — were modified into LM-based forms with improved small-sample properties and placed within a unified asymptotic framework derived from the conditional moment testing principle of Pagan and Hall (1983).

The four modified tests were combined into a composite specification procedure that controls the familywise Type I error rate at any desired level through a Bonferroni correction. A Monte Carlo simulation with 2,000 replications evaluated finite-sample size and power across $n \in \{30, 50, 100, 200\}$ and various misspecification scenarios. Results established that modified tests achieve satisfactory size control for $n \geq 50$, that power increases consistently with sample size, and that the composite procedure achieves substantially higher power than any individual test when multiple misspecifications coexist.

A cross-misspecification analysis documented the size distortions experienced by individual tests when untargeted violations are present, providing empirical support for the composite approach. The JB_mod test was found most sensitive to cross-misspecification from heteroscedasticity. The composite procedure's Bonferroni structure provided effective protection against these contamination effects.

The empirical application to an agricultural production function illustrated how the composite procedure directs model specification by identifying functional form misspecification and heteroscedasticity while clearing the model of serial correlation and non-normality concerns, providing actionable and specific diagnostic guidance.

Future research may extend the framework by replacing the Bonferroni correction with more powerful multiple testing procedures such as the Benjamini-Hochberg false discovery rate control or Holm's step-down procedure. The framework could also be extended to nonlinear regression models, instrumental variables estimation, and panel data structures where spatial and temporal correlation introduce additional diagnostic concerns beyond those addressed in the present study.

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