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Tadpole graphs and their laceability

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ABSTRACT

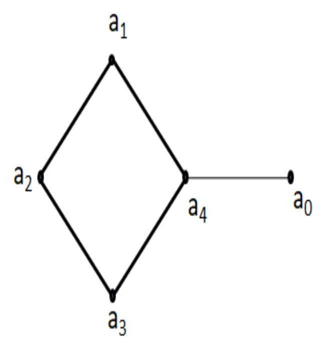
A connected graph G is termed Hamiltonian-t-laceable (t*-laceable) if there exists in G a Hamiltonian path between every pair (at least one pair) of its vertices u and v with the property d(u,v) = t. The Tadpole graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge. In this paper, we discuss the laceability properties associated with the Tadpole graph.

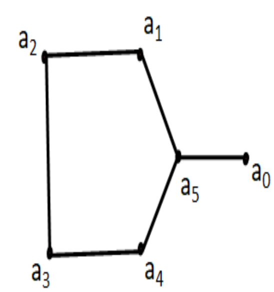
Keywords: Hamiltonian-t-laceability,Hamiltonian-t*-laceability,Tadpole graph

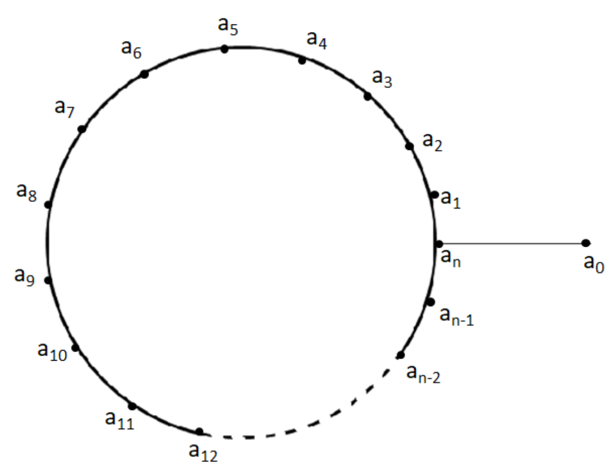
INTRODUCTION

All graphs considered here are finite, simple, connected and undirected. A graph G is hamiltonian-t-laceable (hamiltonian-t*-laceable) if there exists in G a hamiltonian path between every pair (at least one pair)of vertices u and v such that$d(u,v) = t$, $1 \le t \le diamG$, where t is a positive integer. G is termed t*-connected if it is hamiltonian-t*-laceable for all t such that $1 \le t \le diamG$. Various results on the laceability properties of graph structures like the fan graph, the gear graph, the wheel graph, brick product of even and odd cycles and the cyclic product can be found in [5]. In this paper we study the hamiltonian-t*-laceability property of the Tadpole graphs.

Definition: The Tadpole graph is the graph obtained by joining a cycle graph C_m to a path graph P_n with a bridge.







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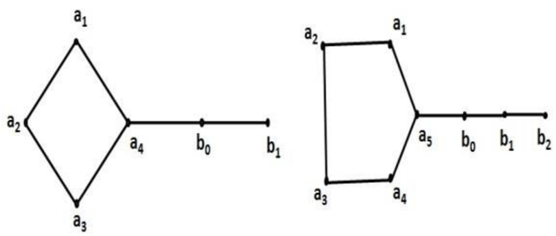


Fig 1: The Tadpole graphs $T_{(4,1)}$, $T_{(5,1)}$, $T_{(n,1)}$, $T_{(4,2)}$ and $T_{(5,3)}$

Definition

Let P be a path from the vertices a_i to a_j in a graph G and let P' be a path from a_j to a_k. Then the path $P \cup P'$ is the path obtained by extending the path P from a_i to a_j to a_k, through the common vertex a_j.

Definition

In a graph $G=(V,E)$, let X be a minimal set of edges not in G and $G' = (V, E \cup X)$ is t^*-connected graph then $|X|$ is called the t^*-laceability number $\lambda^*(t)$ and the edges in X are called the t^*-laceability edges.

RESULTS

We need to introduce the following terminologies.

For a vertex x_i of G, we write:

$x_i^1[n] = x_i, x_{(i+1)}, x_{(i+2)}, \ldots\ldots x_n$.

$x_i^{-1}[m] = x_i, x_{(i-1)}, x_{(i-2)}, \ldots\ldots x_m$.

$\{x_i y_j z_k\}^r [l,m,n] = x_{(i+r)}, y_{(j+r)}, z_{(k+r)}, x_{(i+2r)}, y_{(j+2r)}, z_{(k+2r)} \ldots\ldots x_l, y_m, z_n$.

$x_i^{(l,m,n)} = x_{i+l}, x_{i+m}, x_{i+n}$.

Theorem 1: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 1$ for all t such that $2 \le t \le diam(G)$, where $3 < n \le 6$

Proof: Let $G = T_{(n,1)}$ with vertex set $V = \{a_0, a_1, a_2, a_3, \ldots\ldots, a_n\}$ and edge set $E = \{a_1a_2, a_2a_3, a_3a_4, \ldots\ldots, a_{(n-2)}a_{(n-1)}, a_{(n-1)}a_n\} \cup \{a_n$, where $3 < n \le 6$.

Clearly $diam(G) = 3$.

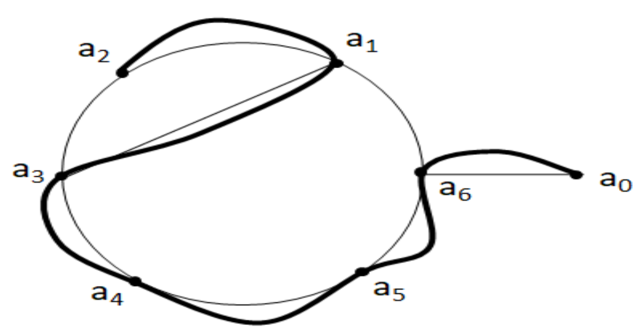
To establish the results, we consider the following cases,

Case (i): $t = 2$

In $G = T_{(n,1)}$, $d(a_1, a_0) = 2$ and the path, $P : a_1 I^1[n] P^0 a_0$ is the hamiltonian path between the vertices a_1 and a_0 containing the laceability edge (a_1, a_3).

Case (ii): $t = 3$

In $G = T_{(n,1)}$, $d(a_2, a_0) = 3$ and the path, $P : a_2 I^{-1} I^2 P^0 I^1 [n] P^0 a_0$ is the hamiltonian path between the vertices a_2 and a_0 containing the laceability edge (a_1, a_3).



Hamiltonian path from vertex a_2 to a_0 in the graph $T_{(6,1)}$

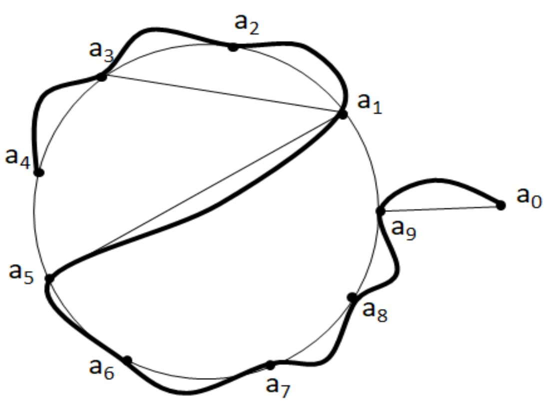
Hence the proof.

Theorem 2: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 1$ for *t=2,3* and with $\lambda_{(t)} = 2$ for *t=4*, where $7 \le n \le 10$.

Proof: Let $G = T_{(n,1)}$

For *t=2,3*, we follow the proof provided in theorem 1. For t=4, we observe that since diam(G)=4,

$d(a_4, a_0) = 4$ in G and the path, $P: a_4 I^{-1}[1] P^0 I^5 P^0 I^1 [n] P^0 a_0$ is ahamiltonian path between the vertices a_4 and a_0 with the laceability edges (a_1, a_3) and (a_1, a_5).



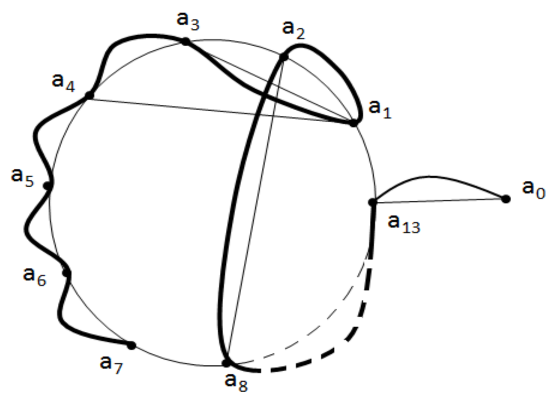
Hamiltonian path from vertex a_4 to a_0 in the graph $T_{(9,1)}$

Hence the proof.

Theorem 3: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 1$ for *t=2,3* with $\lambda_{(t)} = 2$ for *t=4* and with $\lambda_{(t)} = 3$ for *t=5*, where $11 \le n \le 13$.

Proof: Let $G = T_{(n,1)}$

For *t=2,3*, we follow the proof provided in theorem 1. For t=4, we follow the proof provided in theorem 2 and for *t=5*, we observe that since *diam(G)=5*, $d(a_7, a_0) = 5$ in G and the path, $P: a_7 I^{-1}[1] P^0 I^1 P^0 I^6 P^0 I^1 [n] P^0 a_0$ is the hamiltonian path between the vertices a_7 and a_0 with the laceability edges (a_1, a_3), (a_1, a_5) and (a_2, a_8).



Hamiltonian path from vertex a_7 to a_0 in the graph $T_{(13,1)}$

Hence the proof.

The following lemma is an immediate consequence of theorem 3.

Lemma 4: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 4$ for *n=14, where* $2 \le t \le diam(G)$.

Remark: The graph $T_{(n,1)}$ is t*-connected with $\lambda^*(t) = 4$ for $n = 14$

Theorem 5: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 1$ for *t=2,3* with $\lambda_{(t)} = 2$ for *t=4*, with $\lambda_{(t)} = 3$

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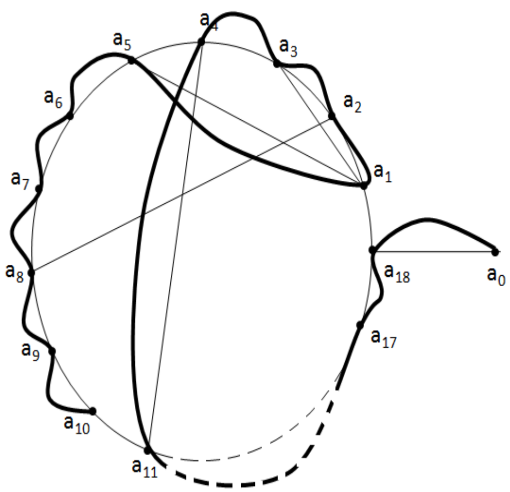
for *t=5* and with $\lambda_{(t)} = 4$ for *t=6*, where $15 \le n \le 18$.

Proof: Let $G = T_{(n,1)}$

For *t=2,3*, we follow the proof provided in theorem 1. For t=4, we follow the proof provided in theorem 2, for *t=5*, we follow the proof provided in theorem 3 and we observe that since *diam(G)=6*, $d(a_{10}, a_0) = 6$ in G and the path, $P: a_{10} I^{-1}[5] P^0 a_1 I^1 [4] P^0 a_{11} I^1 [n] P^0 a_0$ is the hamiltonian path between the vertices a_{10} and a_0 with the laceability edges (a_1, a_3), (a_1, a_5), (a_2, a_8) and (a_4, a_{11}).



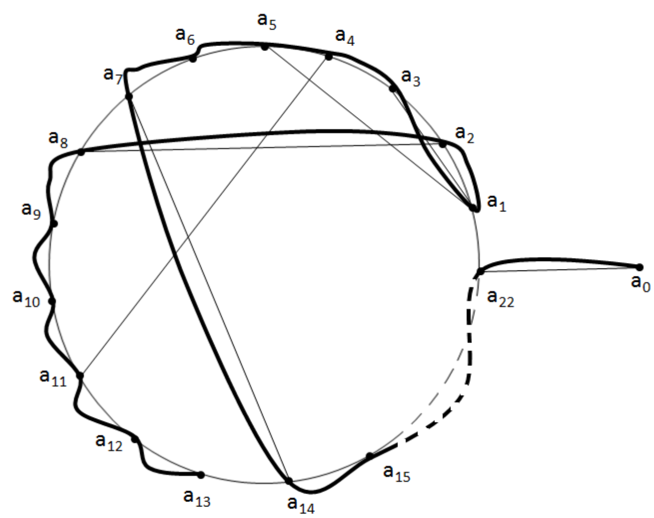
Hamiltonian path from vertex a_{10} to a_0 in the graph $T_{(18,1)}$

Hence the proof.

Theorem 6: The graph $G = T_{(n,1)}$ is hamiltonian-t*-laceable with $\lambda_{(t)} = 1$ for *t=2,3* with $\lambda_{(t)} = 2$ for *t=4*, with $\lambda_{(t)} = 3$ for *t=5*, $\lambda_{(t)} = 4$ for *t=6* and with $\lambda_{(t)} = 5$ for *t=7* where $19 \le n \le 22$..

Proof: Let $G = T_{(n,1)}$

For *t=2,3*, we follow the proof provided in theorem 1. For t=4, we follow the proof provided in theorem 2, for *t=5*, we follow the proof provided in theorem 3, for *t=6*, we follow the proof provided in theorem 5 and for *t=7* we observe that since *diam(G)=7*, $d(a_{13}, a_0) = 7$ in G and the path, $P: a_{13} I^{-1}[8] P^0 a_2 I^{-1} P^0 a_3 I^1 [7] P^0 a_{14} I^1 [n] P^0 a_0$ is the hamiltonian path between the vertices a_{13} and a_0 with the laceability edges (a_1, a_3), (a_1, a_5), (a_2, a_8), (a_4, a_{11}) and (a_7, a_{14}).



Hamiltonian path from vertex a_{13} to a_0 in the graph $T_{(22,1)}$

Hence the proof.

CONCLUSION

In this paper we have explored the hamiltonian-t*-laceability propertiesof tadpolegraphs $T_{(n,1)}$ for $3 < n \le 22$ and $2 \le t \le diam(G)$.

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