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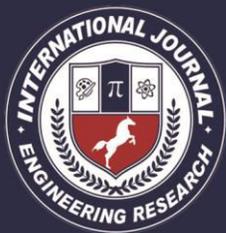
*** S.RAVI KIRAN, P.SNEHALATHA, K.L.N.MURTHY.**

* Eswar College of Engineering.



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DETERMINATION OF FAILURE STRENGTH OF FLATE PLATE WELD JOINT USING FINITE ELEMENT ANALYSIS

¹K. SANDYA SERIN, ²Y.SUVARNA KUMAR, ³K.L.N.MURTHY

¹PG Scholar, Eswar College of engineering, Narasaraopet, Andhra Pradesh.

²Assistant professor, Eswar College of engineering, Narasaraopet, Andhra Pradesh.

³Assistant professor, Eswar College of engineering, Narasaraopet, Andhra Pradesh.

Sandyasherin72@gmail.com, kumar.yarram360@gmail.com, Kandunuri.123@gmail.com

ABSTRACT:

Almost all fabrication of structures today involves welding. Therefore the effects of welding on the life of structures subjected to cyclic loading must be considered for economical and safe design. The vast majority of component fatigue failures take place at the welded connections when the welded structures subjected to fatigue and impact loading.

The aim of this project is the study the effect of the location of welding joint on the fatigue life and fatigue strength of steel plates and finding the best location for welding joint. The scope of this work is applied mechanics and the design of welding joint location.

The welding joints modeling in SOLID WORKS software and analysis in ANSYS software. The welding pieces materials is steel And applying the different loads(60N, 80N, 100N, 120N) on the welded plates for finding the strength, life of the component.

Static analysis to determine the deformation, stress and strain when we applying loads on the component. Fatigue analysis to determine the life, damage and safety factor to estimate the life of the component when applying the repeated cyclic loads.

I.INTRODUCTION

The problem of connecting plates was first solved through riveted connections but the development that occurred during World War-II saw the welded joints replace riveted joints in most applications. The ship building industry was perhaps in the fore front and large ships in excess of 10,000 in number were build with welded structures welding technology, indeed provided several advantages. The ease of processing and weight reduction were the identifiable advantages in the beginning. The automation and variety of welding processes have now become the most obvious advantages the technological developments have included several steels and even non-ferrous metals in the lists of weldable materials.

Fabrication of structural components inherently involves the cases when repairs are

necessary. This is due to the prolonged time of structure operation, and the effect of various phenomena, like corrosion, fatigue, or rheology. Quite often repairs are caused by the events of random nature, but even so, the outcome can be invariably the failure of structure. If this is the case, then the best remedy is to replace the element that has failed with a new one, but due to economic factors and complicated nature of this operation, it is often far easier to fix the problem locally. This study describes a truss that has suffered failure. One of the means used to fix the problem was replacement of the damaged plates and fabrication of a welded joint composed of five plates. To better evaluate this solution, the properties of the joint were examined.

TYPES OF WELDED JOINTS

Two types of welding joints are clearly recognized viz.

- joints between two plates that Overlap and,
- joints between two plates that Butt with each other.

The American Welding Society defines a joint as “the manner in which materials fit together.”

- Butt joint.
- T-joint.
- Lap joint.
- Corner joint.
- Edge joint.

JOINT PREPARATION

Weld joints may be initially prepared in a number of ways. These include:

- Shearing.
- Casting.
- Forging.
- Machining.
- Stamping.
- Filing.
- Routing.
- Oxyacetylene cutting (thermal cutting process).
- Plasma arc cutting (thermal cutting process).
- Grinding.

WELD TYPES

There are various types of welds that can be made in each of the basic joints. They include:

- Square-groove butt weld.
- Bevel-groove butt weld.
- V-groove butt weld.
- J-groove butt weld.
- U-groove butt weld.
- Flare-V-groove butt weld.
- Flare-bevel-groove butt weld.
- Fillet weld.
- Plug weld.

- Slot weld.
- Bevel-groove weld.
- J-groove weld.
- Flare-bevel-groove weld.
- Melt-through weld etc.,

DESIGN OF BUTT WELDS:

For butt welds the most critical form of loading is tension applied in the transverse direction. It has been observed from tests conducted on tensile coupons containing a full penetration butt weld normal to the applied load that the welded joint had higher strength than the parent metal itself. The yield stress of the weld metal and the parent metal in the HAZ region was found to be much higher than the parent metal. The butt weld is normally designed for direct tension or compression. However, a provision is made to protect it from shear. Design strength value is often taken the same as the parent metal strength. For design purposes, the effective area of the butt-welded connection is taken as the effective length of the weld times the throat size. Effective length of the butt weld is taken as the length of the continuous full size weld. The throat size is specified by the effective throat thickness. For a full penetration butt weld, the throat dimension is usually assumed as the thickness of the thinner part of the connection. Even though a butt weld may be reinforced on both sides to ensure full cross-sectional areas, its effect is neglected while estimating the throat dimensions. Such reinforcements often have a negative effect, producing stress concentration, especially under cyclic loads.

DESIGN OF FILLET WELDS:

Fillet welds are broadly classified into side fillets and end fillets. When a connection with end fillet is loaded in tension, the weld develops high strength and the stress developed in the weld is equal to the value of the weld metal. But the ductility is minimal. On the other hand, when a specimen with side weld is

loaded, the load axis is parallel to the weld axis. The weld is subjected to shear and the weld shear strength is limited to just about half the weld metal tensile strength.

DESIGN OF PLUG AND SLOT WELDS:

In certain instances, the lengths available for the normal longitudinal fillet welds may not be sufficient to resist the loads. In such a situation, the required strength may be built up by welding along the back of the channel at the edge of the plate if sufficient space is available. Another way of developing the required strength is by providing slot or plug welds. Slot and plug welds are generally used along with fillet welds in lap joints. On certain occasions, plug welds are used to fill the holes that are temporarily made for erection bolts for beam and Design of Steel Structures. However, their strength may not be considered in the overall strength of the joint.

WELDMENT CONFIGURATIONS

The basic joint often is changed to assist in a component's assembly. A weld joint might be modified to gain access to the weld joint or to change a weld's metallurgical properties. Some common weldment configuration designs are described here. Joggle-type joints are used in cylinder and head assemblies where backing bars or tooling cannot be used. Another application of joggle joints is in the repair of unibody automobiles where skin panels are placed together and welded. A built-in backing bar is used when enough material is available for machining the required backing or when tooling cannot be inserted (as in some tubular applications). Pipe joints often use special backing rings or are machined to fit specially designed mated parts. The fabricated bars must fit tightly or problems will be encountered in heat flow and penetration. Weld joints specially designed for controlled penetration are used where excessive weld penetration would cause a problem with assembly or liquid flow.

A series of bead welds overlaid on the face of a joint is called buttering. Buttered welds are often used to join dissimilar metals. A series of overlaid welds on the surface of a part to protect the base material is called surfacing or cladding.

WELD POSITIONS

For a welder, it is important to be able to weld in different positions. The American Welding Society has defined the positions of welding to include:

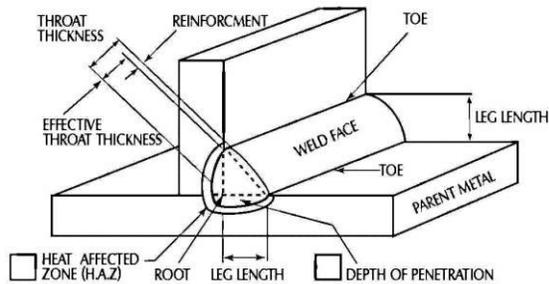
- Flat.
- Horizontal.
- Vertical.
- Overhead.

While doing welding gravity affects the molten weld pools. In addition to this, heat distribution also varies with each position. These factors make the skills needed for each position distinct. Practice is required to produce good welds in all positions.

DESIGN CONSIDERATIONS

Design of the weld type and weld joint to be used is of prime importance if the weldment is to do the intended job. The weld should be made at reasonable cost. Several factors concerning the weld design must be considered:

- Material type and condition (annealed, hardened, tempered).
- Service conditions (pressure, chemical, vibration, shock, wear).
- Physical and mechanical properties of the completed weld and heat-affected zone.
- Preparation and welding cost.
- Assembly configuration and weld access.
- Equipment and tooling.



MATERIAL PROPERTIES

The wide adoption of welding in recent years has required improved control of steel chemistry in order to provide steels that are —weld able, that is steels that can be joined together with sound metal, of adequate strength and ductility, and with minimal metallurgical damage to adjacent parent metal. The relative economy and degree of ease of welding on particular steel is generally termed ‘weld ability’. In ordinary structural steel, the carbon content is the most important factor determining weld ability. Steels with carbon content below 0.1% have a high gas absorption which is a common cause of porous welds. Steels with carbon content 0.3% and more become brittle when cooled rapidly and for a given rate of cooling, brittleness increases with increase in carbon content. Because rapid cooling is prevalent in welding of thick parts, carbon content is particularly important for shapes and plates where thickness in excess of 25.4 mm or 1 38.1 mm can occur⁸.

To secure good quality of welding connections, the design engineer must have some knowledge of the basic principles of welding metallurgy, the thermal effects of welding on base metal and the properties of materials and techniques used in the process.

LITERATURE REVIEW

1.Fatigue of Steel Weldments

The literature dealing with the fatigue of steel weldments has been reviewed and the effect on fatigue strength of testing conditions, weld geometry, weld metal soundness, residual stress and the microstructure of the weld metal and heat-affected zone has been examined. It has been clearly shown that weld geometry is the most important factor in determining the fatigue properties of a weld. For a given weld geometry, the fatigue strength is determined by the severity of the stress concentration at the weld toe or, with the weld reinforcement removed, by the stress concentration at weld metal defects. Different welding processes influence fatigue strength by producing welds with different degrees of surface roughness and weld metal soundness. Residual stress due to welding only affects fatigue strength for alternating loading and under such conditions a moderate increase in fatigue strength is obtained by thermal stress relief. Larger increases in fatigue strength may be obtained by postweld treatments which produce compressive residual stresses, in place of the original tensile stresses, at the weld toe. The microstructures of the weld metal and heat-affected zone have only a minor effect upon the fatigue strength of welds and are usually masked by the much greater effects of weld geometry and weld defects.

2.Fatigue of Aluminum Alloy Welded Joints

A study of aluminum alloy butt, lap and tee welded joints under axial-stress loading and of butt welds under repeated-bending loading (all of thin-gage plate materials) revealed that their fatigue strengths were affected foremost by the geometric characteristics of the joints. The degree of stress concentration and of symmetry with respect to the load axis both contribute to the following order of decreasing axial-stress fatigue strength for the joints investigated: 1. Butt, bead on. 2. Butt, bead off. 3. Tee, double fillet. 4. Lap, double fillet. 5. Lap, single fillet. 6. Tee, single fillet. Only small differences

were found between the last three joints listed, all of which had substantially lower fatigue strengths than the three joints rated above them. The base metal and filler metal alloys apparently had less effect on fatigue strength than the geometric factors. For any given type joint (butt, lap or tee), the weld size and shape were prime factors affecting their fatigue strengths (except bead-off butt welds). The highest fatigue strength for bead-on butt joints was obtained from welds with low-profile reinforcements and high tensile strength. Fillet welds with a convex shape produced lower fatigue strengths for both lap and tee joints. Tee joints with fillet welds substantially oversize and with the welds blending smoothly into base metals had the highest fatigue strength for that type joint.

3. Investigation into the Fatigue Strength of Fillet Welded Assemblies of E-36-4 Steel As a Function of the Penetration of the Weld Subjected to Tensile and Bending Loads.

This study is aimed at evaluating the effect of incomplete penetration on the fatigue behavior of fillet welded assemblies subjected to tension and bending. Its purpose is to evaluate more precisely the conditions of slow extension of microcracks during the initiation stage. The preferred sites for crack initiation at the weld root or the weld toe were determined by using the finite element method. The experimental program, including 120 fatigue tests, was conducted on 10 mm and 30 mm thick E-36-4 steel plates welded with gas metal arc welding (GMAW) and shielded metal arc welding (SMAW) processes. The results evidenced a propagation phase of short cracks, which may represent 30 to 90% of the fatigue life, and the existence of a critical size of incomplete joint penetration below which the incomplete penetration had no significant effect on the fatigue life of fillet welded assemblies subjected to tension and bending. The

numerical calculations, made with the finite element method, have permitted the modeling of the crack propagation paths as a function of the size of incomplete penetration and the determination of the relations $K_I = f(a/t)$ for each zone of failure of the fillet.

4. Study the Effect of Welding Joint Location on the Fatigue Strength and Fatigue Life for Steel Weldment

The welding process is one of the oldest joining processes between the materials, this paper try to find the effect of welding joint location in the steel on the fatigue strength of steel. The welding process done by electrical arc welding to joining steel samples at different locations at ($X/L=0.25$, $X/L=0.5$, and $X/L=0.75$), where (X) the location of welding zone centre and sample subjected to fully reversed bending stress, then comparing the fatigue test results with un-welded sample. The experimental results show that the welding joint decrease the tensile strength of steel and the fatigue failure strength also decreased specially for that with ($X/L=0.5$ and $X/L=0.75$) and failure occur at welding zone, but the sample with ($X/L=0.25$) had less effected by welding joint and the failure occur at the support not at welding zone. The results show fatigue life affected by the welding joint when draw (S-N) diagram for each sample especially for sample with ($X/L=0.5$ and $X/L=0.75$). Keywords: welding of steel, fatigue, S-N diagram, finite element analysis, fatigue behavior of steel weldment.

INTRODUCTION TO CAD

Computer-aided design (CAD) is the use of computer systems (or workstations) to aid in the creation, modification, analysis, or optimization of a design. CAD software is used to increase the productivity of the designer, improve the quality of design, improve communications through documentation, and to create a database for manufacturing. CAD output is often in the form of electronic files for



print, machining, or other manufacturing operations. The term CADD (for Computer Aided Design and Drafting) is also used.

Its use in designing electronic systems is known as electronic design automation, or EDA. In mechanical design it is known as mechanical design automation (MDA) or computer-aided drafting (CAD), which includes the process of creating a technical drawing with the use of computer software. CAD may be used to design curves and figures in two-dimensional (2D) space; or curves, surfaces, and solids in three-dimensional (3D) space.

CAD is an important industrial art extensively used in many applications, including automotive, shipbuilding, and aerospace industries, industrial and architectural design, prosthetics, and many more. CAD is also widely used to produce computer animation for special effects in movies, advertising and technical manuals, often called DCC digital content creation. The modern ubiquity and power of computers means that even perfume bottles and shampoo dispensers are designed using techniques unheard of by engineers of the 1960s. Because of its enormous economic importance, CAD has been a major driving force for research in computational geometry, computer graphics (both hardware and software), and discrete differential geometry.

INTRODUCTION TO SOLID WORKS

Solid Works (stylized as SOLIDWORKS) is a strong modeling computer-aided layout (CAD) and laptop-aided engineering (CAE) computer application that runs on Microsoft Windows. Solid Works is published with the aid of Dassault Systems.

According to the writer, over million engineers and architects at extra than 165,000 corporations were the use of Solid Works as of 2013. Also in line with the organization, financial year 2011–12 revenue for Solid Works totaled \$483 million.

DS Solid works Corp. Has sold over 1.5 million licenses of Solid Works global. This consists of a massive proportion of tutorial licenses. The Sheffield Telegraph feedback that Solidworks is the world's most famous CAD software program. Its user base stages from individuals to large agencies, and covers a totally huge move-section of manufacturing market segments. Commercial income are made thru an indirect channel, which incorporates dealers and partners at some stage in the arena. In the United States, the primary reseller of Solid Works, in 1995, changed into Computer Aided Technology, Inc, situated in Chicago. Directly competitive products to Solid Works encompass Solid Edge, and Autodesk Inventor. Solid Works additionally companions with 1/3 birthday celebration developers to feature capability in area of interest market programs like finite element evaluation, circuit format, tolerance checking, and so on. Solid Works has also certified its 3D modeling competencies to other CAD software program carriers, significantly ANVIL.

INTRODUCTION TO FEA

Finite element analysis is a method of solving, usually approximately, certain problems in engineering and science. It is used mainly for problems for which no exact solution, expressible in some mathematical form, is available. As such, it is a numerical rather than an analytical method. Methods of this type are needed because analytical methods cannot cope with the real, complicated problems that are met with in engineering. For example, engineering strength of materials or the mathematical theory of elasticity can be used to calculate analytically the stresses and strains in a bent beam, but neither will be very successful in finding out what is happening in part of a car suspension system during cornering.

One of the first applications of FEA was, indeed, to find the stresses and strains in engineering components under load. FEA,

when applied to any realistic model of an engineering component, requires an enormous amount of computation and the development of the method has depended on the availability of suitable digital computers for it to run on. The method is now applied to problems involving a wide range of phenomena, including vibrations, heat conduction, fluid mechanics and electrostatics, and a wide range of material properties, such as linear-elastic (Hookean) behavior and behavior involving deviation from Hooke's law (for example, plasticity or rubber-elasticity).

INTRODUCTION TO ANSYS

Structural Analysis

ANSYS Autodyn is computer simulation tool for simulating the response of materials to short duration severe loadings from impact, high pressure or explosions.

ANSYS Mechanical

ANSYS Mechanical is a finite element analysis tool for structural analysis, including linear, nonlinear and dynamic studies. This computer simulation product provides finite elements to model behavior, and supports material models and equation solvers for a wide range of mechanical design problems. ANSYS Mechanical also includes thermal analysis and coupled-physics capabilities involving acoustics, piezoelectric, thermal-structural and thermo-electric analysis.

Fluid Dynamics

ANSYS Fluent, CFD, CFX, FENSAP-ICE and related software are Computational Fluid Dynamics software tools used by engineers for design and analysis. These tools can simulate fluid flows in a virtual environment — for example, the fluid dynamics of ship hulls; gas turbine engines (including the compressors, combustion chamber, turbines and afterburners); aircraft aerodynamics; pumps,

fans, HVAC systems, mixing vessels, hydro cyclones, vacuum cleaners, etc.

STATIC ANALYSIS OF WELDED JOINTS

Definition of Static Analysis

A static analysis calculates the effects of steady loading conditions on a structure, while ignoring inertia and damping effects, such as those caused by time-varying loads. A static analysis can, however, include steady inertia loads (such as gravity and rotational velocity), and time-varying loads that can be approximated as static equivalent loads (such as the static equivalent wind and seismic loads commonly defined in many building codes).

Loads in a Static Analysis

Static analysis is used to determine the displacements, stresses, strains, and forces in structures or components caused by loads that do not induce significant inertia and damping effects. Steady loading and response conditions are assumed; that is, the loads and the structure's response are assumed to vary slowly with respect to time. The kinds of loading that can be applied in a static analysis include:

- Externally applied forces and pressures
- Steady-state inertial forces (such as gravity or rotational velocity)
- Imposed (non-zero) displacements
- Temperatures (for thermal strain)
- Fluxes (for nuclear swelling)

Load Types

Displacements (UX, UY, UZ, ROTX, ROTY, ROTZ)

These are DOF constraints usually specified at model boundaries to define rigid support points. They can also indicate symmetry boundary conditions and points of known motion. The directions implied by the labels are in the nodal coordinate system.

Forces (FX, FY, FZ) and moments (MX, MY, MZ)

These are concentrated loads usually specified on the model exterior. The directions implied by the labels are in the nodal coordinate system.

Pressures (PRES)

These are surface loads, also usually applied on the model exterior. Positive values of pressure act towards the element face (resulting in a compressive effect).

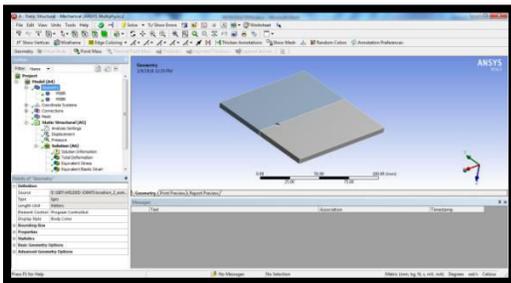
Gravity, spinning, etc.

These are inertia loads that affect the entire structure. Density (or mass in some form) must be defined if inertia effects are to be included.

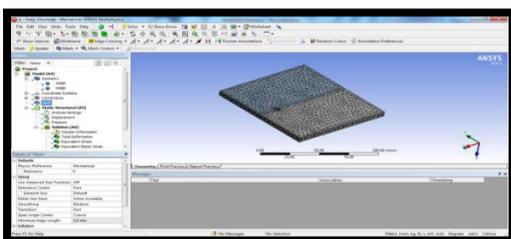
FINITE ELEMENT ANALYSIS OF WELDING JOINT

The model of chassis is saved in IGES format which can be directly imported into ANSYS workbench. The model imported to ANSYS workbench

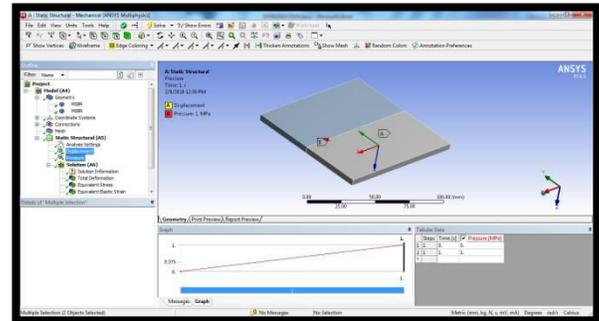
Imported model



Meshed Mode



Boundary Condition

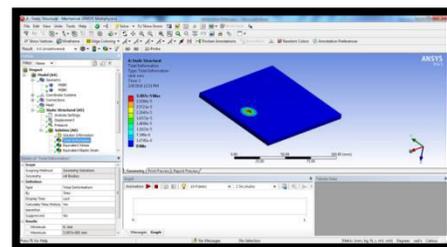


SELECTION OF MATERIAL PROPERTIES

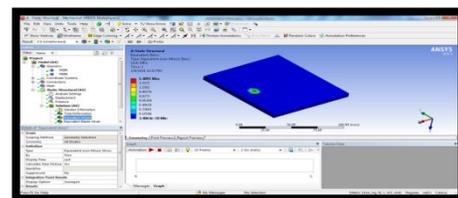
Properties	steel
Density(g/cm ³)	7.85
Young's modulus (MPa)	205000
Poisson's ratio	0.3

MATERIAL-STEEL AT LOAD-60N

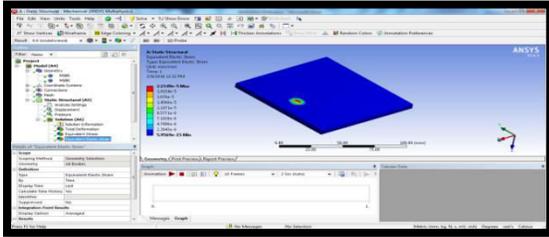
TOTAL DEFORMATION



STRESS

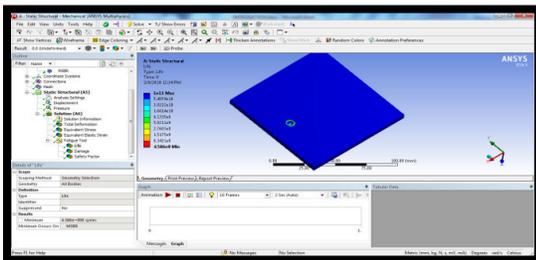


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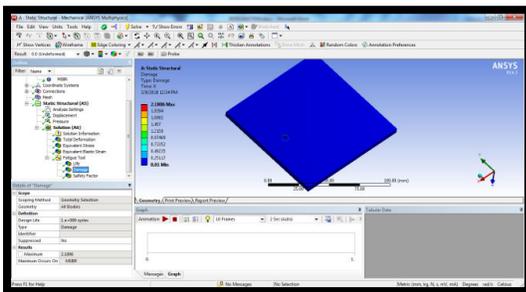


FATIGUE ANALYSIS OF WELDED JOINT

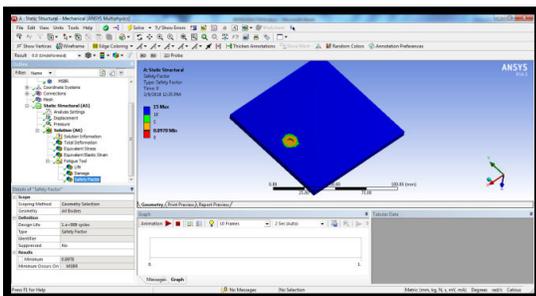
LIFE



DAMAGE

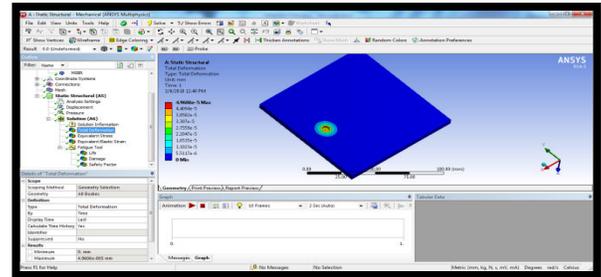


SAFETY FACTOR

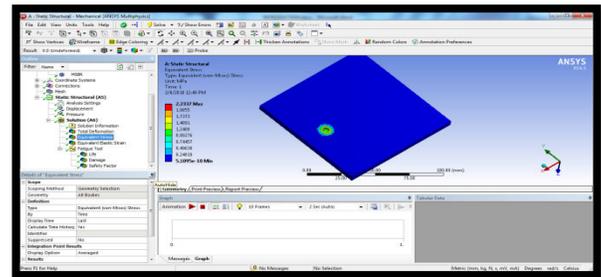


AT LOAD-80N

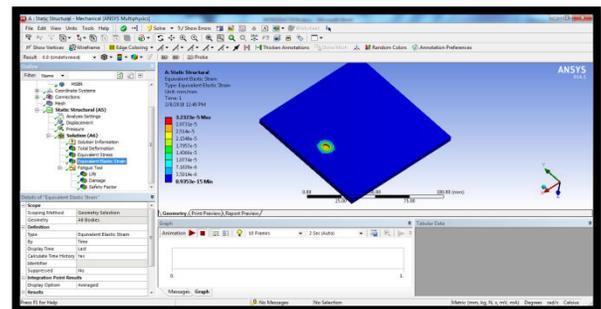
TOTAL DEFORMATION



STRESS

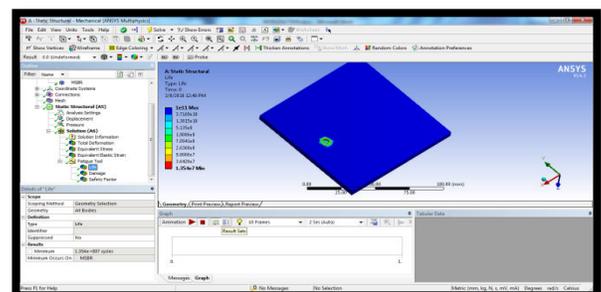


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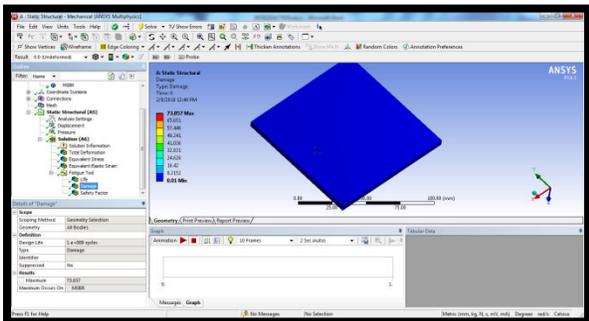


FATIGUE ANALYSIS OF WELDED JOINT

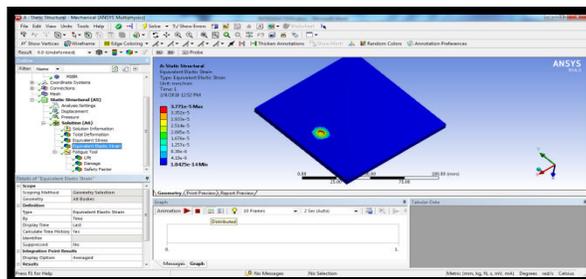
LIFE



DAMAGE

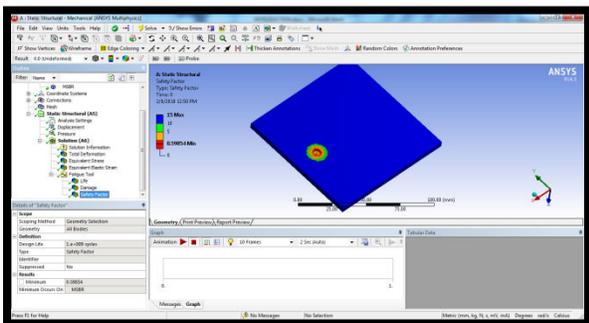


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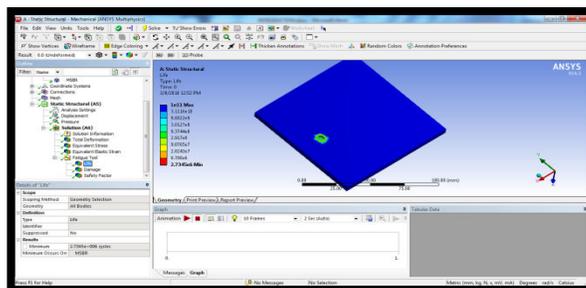


SAFETY FACTOR

FATIGUE ANALYSIS OF WELDED JOINT

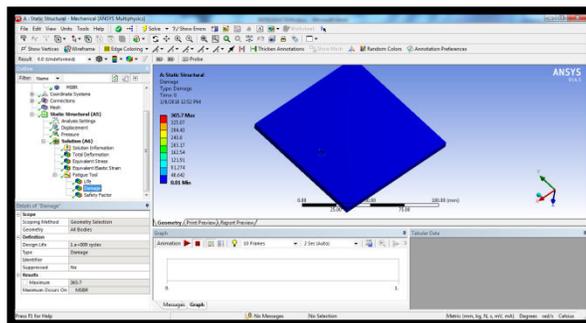
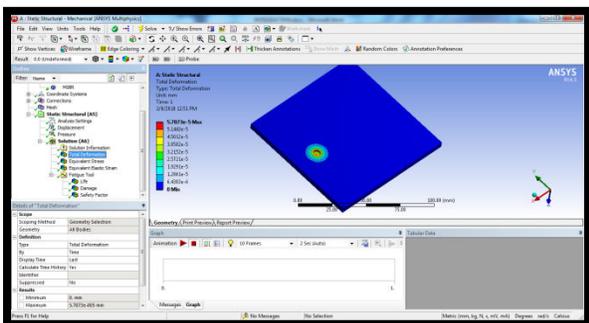


LIFE



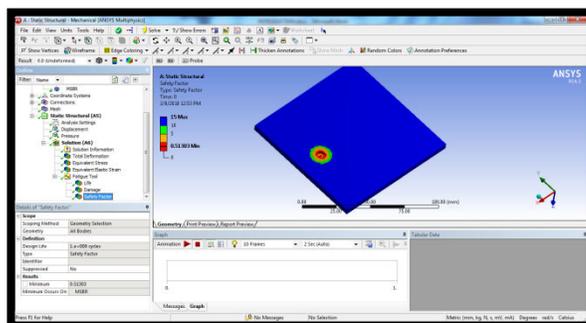
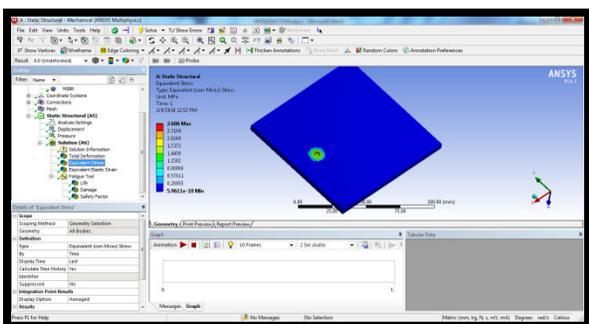
AT LOAD-100N
TOTAL DEFORMATION

DAMAGE

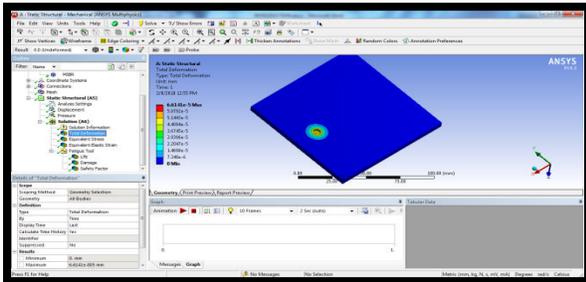


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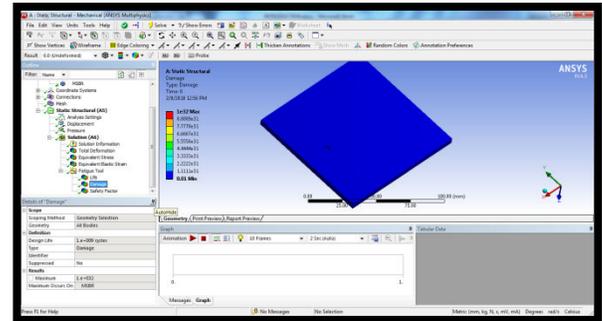
SAFETY FACTOR



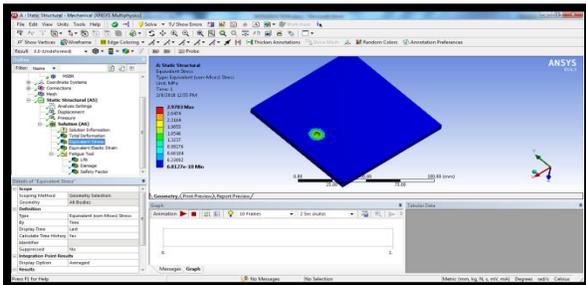
AT LOAD-120N TOTAL DEFORMATION



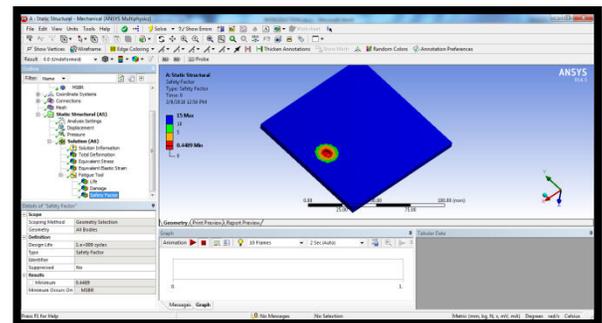
DAMAGE



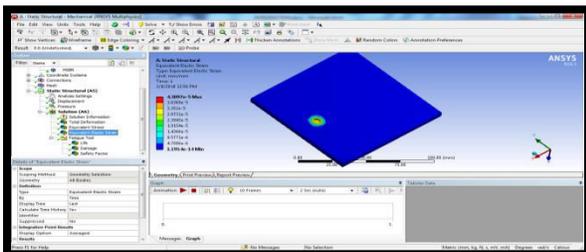
STRESS



SAFETY FACTOR



STRAIN



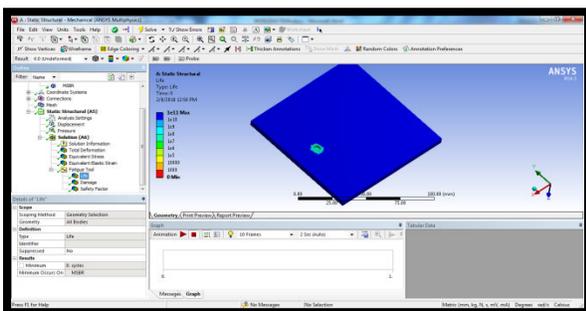
RESULT TABLES

STATIC ANALYSIS RESULTS

At load (N)	Deformation (mm)	Stress (N/mm ²)	Strain
60	3.307e-5	1.4981	2.15e-5
80	4.960e-5	2.237	3.23e-5
100	5.7873e-5	2.606	3.77e-5
120	6.6141e-5	2.9783	4.30e-5

FATIGUE ANALYSIS OF WELDED JOINT

LIFE



FATIGUE ANALYSIS RESULTS

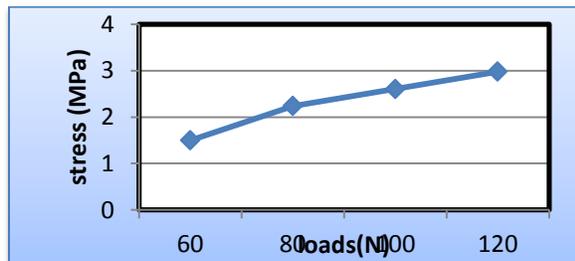
At load (N)	Life	Damage	Safety factor	
			Min	Max
60	1e-11	2.1806	0.8978	15
80	1e-11	73.857	0.5984	15
100	1e-11	365.7	0.51303	15
120	1e-11	412.5	0.4489	15

GRAPHS

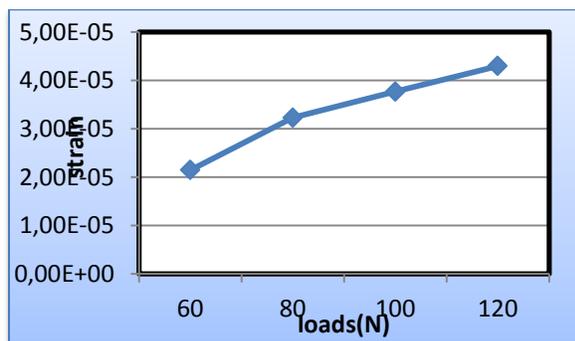
DEFORMATION PLOT



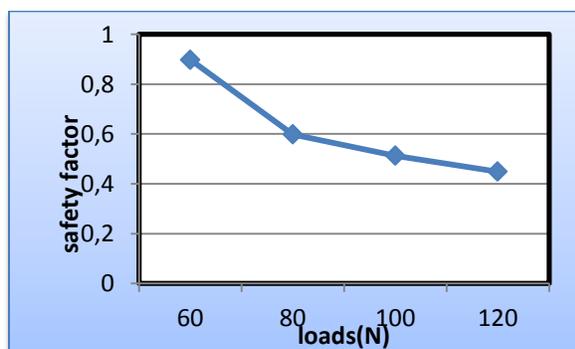
STRESS PLOT



STRAIN PLOT



SAFETY FACTOR



V.CONCLUSION

From the above results and discussion, Knuckle joint was design for 100N and 110N axial load by theoretical calculation. Final dimensions from theoretical calculation, model of Knuckle joint is made in CREO.

In this project ,static analysis done at different loads(100N and 110N) with different materials(steel and cast iron) in static analysis, observed the stress values are less steel compared with cast iron at 100N load.

so it can be concluded that the better material for steel The welding joints modeling in SOLID WORKS software and analysis in ANSYS software. The welding pieces materials is steel and steel replace with dissimilar material plates like aluminum alloy. And applying the different loads (60N, 80N, 100N, 120N) on the welded plates for finding the strength, life of the component.

Almost all fabrication of structures today involves welding. Therefore the effects of welding on the life of structures subjected to cyclic loading must be considered for economical and safe design. The vast majority of component fatigue failures take place at the welded connections when the welded structures subjected to fatigue and impact loading.

By observing the static analysis the stress, deformation and strain values are increases by increasing the loads. Minimum stress value at 60 N load.

By observing the fatigue analysis the safety factor increases by decreasing the loads. safety factor maximum value at 60N load.

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